MVE136 Random Signals Analysis

Written home exam Monday 16 August 2021 2–6 PM

TEACHER AND EXAMINER: Patrik Albin 031 7723512 palbin@chalmers.se .

AIDS: All aids are permitted. (See the Canvas course "Omtentamen 2 Modul: 0111, MVE136" with instructions for this exam for clarifications.)

GRADES: 12 (40%), 18 (60%) and 24 (80%) points for grade 3, 4 and 5, respectively.

MOTIVATIONS: All answers/solutions must be motivated. GOOD LUCK!

Task 1. Find a WSS continuous time random process $\{X(t)\}_{t\in\mathbb{R}}$ that is not strict sense stationary. **(5 points)**

Task 2. Calculate $\Pr(X(0) = 0)$ for a zero-mean WSS random process $X(t)$ with autocorrelation function $R_{XX}(\tau) = 0$. **(5 points)**

Task 3. Calculate $\Pr(X(1)X(2)X(3)=6)$ for a Poisson process $\{X(t)\}_{t\geq 0}$ with arrival rate 1. **(5 points)**

Task 4. A Markov chain has four states $\{0, 1, 2, 3\}$ and all transition probabilities $p_{ij} = 1/4$. Calculate the expected value of the time it takes for the chain to move from state 0 to state 3. **(5 points)**

Task 5. You may select any zero-mean WSS process $\{X(t)\}_{t\in\mathbb{R}}$ as insignal to an LTI system with impulse response $h(t) = \text{sinc}(t) = \sin(\pi t)/(\pi t)$ for $t \in \mathbb{R}$. What are the restrictions on the possible outsignals $\{Y(t)\}_{t\in\mathbb{R}}$ from the LTI system? **(5 points)**

Task 6. Explain why an AR(1)-process is a Markov chain. **(5 points)**

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Solutions to written exam 16 August 2021

Task 1. Let all values of $X(t)$ be independent of each other with zero-mean and unit variance but not all CDF's $F_{X(t)}(x)$ being the same.

Task 2. 1.

Task 3. Clearly $\Pr(X(1)X(2)X(3) = 6) = \Pr(X(1) = 1,\, X(2) = 1,\, X(3) = 6) + \Pr(X(1) = 1,\, X(2) = 2,\, X(3) = 3) = \Pr(X(1) = 1,\, X(2)-X(1) = 0,\, X(3)-X(2) = 5) + \Pr(X(1)=1,\, X(2)-X(1)=1,\, X(3)-X(2)=1) = \Pr(X(1)=1)\Pr(X(2)-X(1) = 0)\Pr(X(3)-X(2) = 5) + \Pr(X(1) = 1)\Pr(X(2)-X(1) = 1)\Pr(X(3)-X(2) = 1) = \frac{1}{5!}\,(\mathrm{e}^{-1})^3 + (\mathrm{e}^{-1})^3 = \frac{121}{120}\,\mathrm{e}^{-3}$.

Task 4. For the sought after expectation E we have $E = 1 + (3/4)\cdot E$ giving $E = 4$.

Task 5. As the insignal is zero-mean $\mu_X = 0$, so must be the outsignal $\mu_Y = 0$. As for the autocorrelation function $R_{YY}(\tau) = \int_{-\infty}^{\infty} \mathrm{e}^{j2\pi f\tau} S_{YY}(f)\,df$, since $H(f) = 1$ for $|f| \leq 1/2$ and 0 for $|f| > 1/2$ we get $S_{YY}(f) = |H(f)|^2 S_{XX}(f) = 0$ for $|f| > 1/2$ with no other restrictions on $S_{YY}(f)$ (except symmetry and positivity).

Task 6. Because the next value of the process $X[k{+}1] = a\,X[k] + e_{k+1}$ depends on the history of the process $\{X[\ell]\}_{\ell=-\infty}^{k}$ through the last member of the history $X[k]$ only (as e_{k+1} is independent of that history).