Exam in mathematical statistics, Statistical Quality Control (MVE-145/MSG-600)

Tuesday 2007-12-18, 8.30-12.30

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Grade limits GU: **G**: 12-21.5, **VG**: 22-30
Grade limits CTH: **3**: 12-17.5, **4**: 18-23.5, **5**: 24-30

Aid problems 1-2: None. Hand-in separately
Aid problems 3-8: One ***handwritten*** A4-page, tables (Beta or similar) and approved calculator (Chalmersgodkänd räknare)

Langugages: You can choose to write in English or in Swedish.
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Uppgift 1 (3.5 p)

Describe shortly the seven major tools within Statistical process control ("The magnificant seven").
(0.5p per tool).

Uppgift 2 (4.5p)

Answer following question

1. What is the main purpose of a control chart?
2. What is the difference between Phase I and Phase II control charts?
3. What is systematic variation and what is random variation?
4. What is G R&R and how can it be measured?
5. What is OCAP and why is it important?
6. A process characteristics is normally distributed $N(\mu,\sigma)$ and a Shewhart control chart with control limits $\mu \pm 3\sigma$ is used. If the process is in control, what is the risk of false alarms?
7. What is ARL and what will ARL be in question 6?
8. What are the conditions for using capability indices C_p and C_{pk} ?
9. Is this examination a useful model of your knowledge within SPC? Explain.

(0.5p per question)

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Hand in part I
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Problem 3 (3p)

You are responsible for purchase quality on a company manufacturing medical devices. Your company buys needles in batches of size 7600. The contract with your supplier states that acceptance sampling according to MIL STD 105E shall be used with AQL=0.40% and General Inspection Level II.

a) Describe how the sampling will be done and how the test plans looks like. (1p)
b) When should acceptance sampling be used and when should all-control be used? (1p)
c) If you take a sample, and is in "Normal Inspection" and you get 0 defect needles, what can you say about the quality of batch? What is the upper 95% confidence limit on the fraction defective? (1p)

Problem 4 (5p)

Samples of size n=5 are taken from a process every half hour. After m=25 samples have been collected, we calculate

$$\sum_{i=1}^{m} \bar{x}_i = 662.5$$

$$\sum_{i=1}^{m} R_i = 9.00$$

A Shewhart control chart is used. Assume that both charts indicates that the process is in control and that the quality characteristics is independent and normally distributed.

a) Estimate the process standard deviation and estimate the natural tolerance limits. (1p)
b) Find the control limits of the $\bar{x}$ and R charts. (1p)
c) Assume that both both charts exhibit control. If the specification limits are 26.40 ± 0.50, estimate the fraction non-conforming. (1p)
d) If the process mean is adjusted to 26.40, what will ATS (average time to signal) for the $\bar{x}$-chart be? (2p)

Problem 5 (4p) Euler buckling

A beam is free to rotate in both its ends, see Figure 1, can at a certain compressive loads be bent or buckled. This happens at a critical load $P_{crit.}$.

The critical load depends on

- The length of the beam, L
- The E-modulus of the material, E
- The moment of intertia I.

From elementary courses in mechanics of solids we know (solving an eigenvalue problem) that the lowest critcial load (n=1) is

$$P_{crit.} = \frac{\pi^2 EI}{L^2}$$

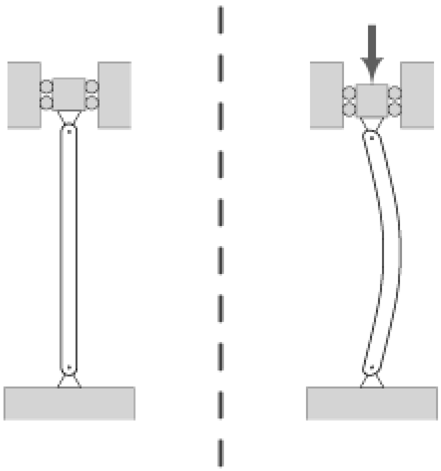


Figure 1 Euler buckling type I. (picture from Wikipedia)

Let us asssume that L, E and I are normally distributed stochastic variables.

$$L \sim N(\mu_L, \sigma_L) = N(0.45, 0.001)m$$
$$E \sim N(\mu_E, \sigma_E) = N(208, 5)GPa$$
$$I \sim N(\mu_I, \sigma_I) = N(5 \cdot 10^{-8}, 1 \cdot 10^{-9})m^4$$

The tolerance limit for the critical load is $\left[4.7 \cdot 10^5, 6.1 \cdot 10^5\right] kN$.

a. Calculate the mean and variance for the critical load $P_{crit.}$ (2p)
b. Which of the three variables E, I and L would you be most interested in if you want to reduce the variation in critical load? (1p)
c. What is your opinion, is the process capable? Motivate! (1p)

Problem 6 (3p)

A $\bar{x}$ -control chart with 3 sigma control limits has following parameters:

$$UCL = 104$$
$$CL = 100$$
$$LCL = 96$$
$$n = 5$$

Suppose that the measured characteristics is normally distributed and has a true average of 98 and standard deviation of 8.
What is the probability that the control chart will raise an out of control alarm after maximum 3 samples?

Uppgift 7 (4p)

The plastic flow in a molding machine is measued every 5 minute. The target for the process is $\mu_0 = 500$. In Table 1 20 measurements are shown.

Sample no	Measurement x_i	Range $MR_i = \lvert x_i - x_{i-1} \rvert$
1	473	
2	512	39
3	518	6
4	492	26
5	484	8
6	512	28
7	513	1
8	536	23
9	481	55
10	533	52
11	536	3
12	538	2
13	539	1
14	523	16
15	535	12
16	513	22
17	553	40
18	544	9
19	585	41
20	527	58
	$\sum x_i = 10477$	$\sum MR_i = 442$

Table 1

a. Estimate the process standard deviation. (1p)
b. Design a EWMA diagram that controls the process average and determine whether the process is in control or not. (Use $\lambda = 0.1$ and L=2.7). Is the process in control? (3p)

Uppgift 8 (3p)

Assume that a process is in control, has normally distributed and independent outcome. Let p be the fraction products outside the tolerance limits.

a. Write a formula for p, that depends on the capability indices C_p and C_{pk} : $p = f(C_p, C_{pk})$ and calculate p for $C_p = 1.67$ and $C_{pk} = 1.33$. (2p)
b. If the process is not in control, what is then your conclusion about p? (1p)

Poisson table

	λ						
x	1	2	3	4	5	6	7
0	0.36788	0.13534	0.04979	0.01832	0.00674	0.00248	0.00091
1	0.73576	0.40601	0.19915	0.09158	0.04043	0.01735	0.00730
2	0.91970	0.67668	0.42319	0.23810	0.12465	0.06197	0.02964
3	0.98101	0.85712	0.64723	0.43347	0.26503	0.15120	0.08177
4	0.99634	0.94735	0.81526	0.62884	0.44049	0.28506	0.17299
5	0.99941	0.98344	0.91608	0.78513	0.61596	0.44568	0.30071
6	0.99992	0.99547	0.96649	0.88933	0.76218	0.60630	0.44971
7	0.99999	0.99890	0.98810	0.94887	0.86663	0.74398	0.59871
8	1	0.99976	0.99620	0.97864	0.93191	0.84724	0.72909
9	1	0.99995	0.99890	0.99187	0.96817	0.91608	0.83050
10	1	0.99999	0.99971	0.99716	0.98630	0.95738	0.90148
11	1	1	0.99993	0.99908	0.99455	0.97991	0.94665
12	1	1	0.99998	0.99973	0.99798	0.99117	0.97300
13	1	1	1	0.99992	0.99930	0.99637	0.98719
14	1	1	1	0.99998	0.99977	0.99860	0.99428
15	1	1	1	1	0.99993	0.99949	0.99759

Normal distribution

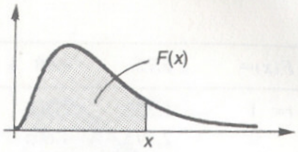
	0	1	2	3	4	5	6	7	8	9
0.00	0.500	0.504	0.508	0.512	0.516	0.520	0.524	0.528	0.532	0.536
0.10	0.540	0.544	0.548	0.552	0.556	0.560	0.564	0.567	0.571	0.575
0.20	0.579	0.583	0.587	0.591	0.595	0.599	0.603	0.606	0.610	0.614
0.30	0.618	0.622	0.626	0.629	0.633	0.637	0.641	0.644	0.648	0.652
0.40	0.655	0.659	0.663	0.666	0.670	0.674	0.677	0.681	0.684	0.688
0.50	0.691	0.695	0.698	0.702	0.705	0.709	0.712	0.716	0.719	0.722
0.60	0.726	0.729	0.732	0.736	0.739	0.742	0.745	0.749	0.752	0.755
0.70	0.758	0.761	0.764	0.767	0.770	0.773	0.776	0.779	0.782	0.785
0.80	0.788	0.791	0.794	0.797	0.800	0.802	0.805	0.808	0.811	0.813
0.90	0.816	0.819	0.821	0.824	0.826	0.829	0.831	0.834	0.836	0.839
1.00	0.841	0.844	0.846	0.848	0.851	0.853	0.855	0.858	0.860	0.862
1.10	0.864	0.867	0.869	0.871	0.873	0.875	0.877	0.879	0.881	0.883
1.20	0.885	0.887	0.889	0.891	0.893	0.894	0.896	0.898	0.900	0.9015
1.30	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.40	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.50	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.60	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.70	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.80	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.90	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.00	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.10	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.20	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.30	0.9893	0.9896	0.9898	$0.9^{2}001$	$0.9^{2}036$	$0.9^{2}061$	$0.9^{2}086$	$0.9^{2}111$	$0.9^{2}134$	$0.9^{2}158$
2.40	$0.9^{2}180$	$0.9^{2}202$	$0.9^{2}224$	$0.9^{2}245$	$0.9^{2}266$	$0.9^{2}286$	$0.9^{2}305$	$0.9^{2}324$	$0.9^{2}343$	$0.9^{2}361$
2.50	$0.9^{2}379$	$0.9^{2}396$	$0.9^{2}413$	$0.9^{2}430$	$0.9^{2}446$	$0.9^{2}461$	$0.9^{2}477$	$0.9^{2}492$	$0.9^{2}506$	$0.9^{2}520$
2.60	$0.9^{2}534$	$0.9^{2}547$	$0.9^{2}560$	$0.9^{2}573$	$0.9^{2}585$	$0.9^{2}598$	$0.9^{2}609$	$0.9^{2}621$	$0.9^{2}632$	$0.9^{2}643$
2.70	$0.9^{2}653$	$0.9^{2}664$	$0.9^{2}674$	$0.9^{2}683$	$0.9^{2}693$	$0.9^{2}702$	$0.9^{2}711$	$0.9^{2}720$	$0.9^{2}728$	$0.9^{2}736$
2.80	$0.9^{2}744$	$0.9^{2}752$	$0.9^{2}760$	$0.9^{2}767$	$0.9^{2}774$	$0.9^{2}781$	$0.9^{2}788$	$0.9^{2}795$	$0.9^{2}801$	$0.9^{2}807$
2.90	$0.9^{2}813$	$0.9^{2}819$	$0.9^{2}825$	$0.9^{2}831$	$0.9^{2}836$	$0.9^{2}841$	$0.9^{2}846$	$0.9^{2}851$	$0.9^{2}856$	$0.9^{2}861$
3.00	$0.9^{2}865$	$0.9^{2}869$	$0.9^{2}874$	$0.9^{2}878$	$0.9^{2}882$	$0.9^{2}886$	$0.9^{2}889$	$0.9^{2}893$	$0.9^{2}896$	$0.9^{2}900$
3.10	$0.9^{3}032$	$0.9^{3}065$	$0.9^{3}096$	$0.9^{3}126$	$0.9^{3}155$	$0.9^{3}184$	$0.9^{3}211$	$0.9^{3}238$	$0.9^{3}264$	$0.9^{3}289$
3.20	$0.9^{3}313$	$0.9^{3}336$	$0.9^{3}359$	$0.9^{3}381$	$0.9^{3}402$	$0.9^{3}423$	$0.9^{3}443$	$0.9^{3}462$	$0.9^{3}481$	$0.9^{3}499$
3.30	$0.9^{3}517$	$0.9^{3}534$	$0.9^{3}550$	$0.9^{3}566$	$0.9^{3}581$	$0.9^{3}596$	$0.9^{3}610$	$0.9^{3}624$	$0.9^{3}638$	$0.9^{3}651$
3.40	$0.9^{3}663$	$0.9^{3}675$	$0.9^{3}687$	$0.9^{3}698$	$0.9^{3}709$	$0.9^{3}720$	$0.9^{3}730$	$0.9^{3}740$	$0.9^{3}749$	$0.9^{3}758$

z	0	1	2	3	4	5	6	7	8	9
3.50	$0.9^{3}767$	$0.9^{3}776$	$0.9^{3}784$	$0.9^{3}792$	$0.9^{3}800$	$0.9^{3}807$	$0.9^{3}815$	$0.9^{3}822$	$0.9^{3}828$	$0.9^{3}835$
3.60	$0.9^{3}841$	$0.9^{3}847$	$0.9^{3}853$	$0.9^{3}858$	$0.9^{3}864$	$0.9^{3}869$	$0.9^{3}874$	$0.9^{3}879$	$0.9^{3}883$	$0.9^{3}888$
3.70	$0.9^{3}892$	$0.9^{3}896$	$0.9^{4}000$	$0.9^{4}043$	$0.9^{4}080$	$0.9^{4}116$	$0.9^{4}150$	$0.9^{4}184$	$0.9^{4}216$	$0.9^{4}247$
3.80	$0.9^{4}277$	$0.9^{4}305$	$0.9^{4}333$	$0.9^{4}359$	$0.9^{4}385$	$0.9^{4}409$	$0.9^{4}433$	$0.9^{4}456$	$0.9^{4}478$	$0.9^{4}499$
3.90	$0.9^{4}519$	$0.9^{4}539$	$0.9^{4}557$	$0.9^{4}575$	$0.9^{4}593$	$0.9^{4}609$	$0.9^{4}625$	$0.9^{4}641$	$0.9^{4}655$	$0.9^{4}670$
4.00	$0.9^{4}683$	$0.9^{4}696$	$0.9^{4}709$	$0.9^{4}721$	$0.9^{4}733$	$0.9^{4}744$	$0.9^{4}755$	$0.9^{4}765$	$0.9^{4}775$	$0.9^{4}784$
4.10	$0.9^{4}793$	$0.9^{4}802$	$0.9^{4}811$	$0.9^{4}819$	$0.9^{4}826$	$0.9^{4}834$	$0.9^{4}841$	$0.9^{4}848$	$0.9^{4}854$	$0.9^{4}861$
4.20	$0.9^{4}867$	$0.9^{4}872$	$0.9^{4}878$	$0.9^{4}883$	$0.9^{4}888$	$0.9^{4}893$	$0.9^{4}898$	$0.9^{5}023$	$0.9^{5}066$	$0.9^{5}107$
4.30	$0.9^{5}146$	$0.9^{5}184$	$0.9^{5}220$	$0.9^{5}254$	$0.9^{5}288$	$0.9^{5}319$	$0.9^{5}350$	$0.9^{5}379$	$0.9^{5}407$	$0.9^{5}433$
4.40	$0.9^{5}459$	$0.9^{5}483$	$0.9^{5}506$	$0.9^{5}529$	$0.9^{5}550$	$0.9^{5}571$	$0.9^{5}590$	$0.9^{5}609$	$0.9^{5}627$	$0.9^{5}644$
4.50	$0.9^{5}660$	$0.9^{5}676$	$0.9^{5}691$	$0.9^{5}705$	$0.9^{5}719$	$0.9^{5}732$	$0.9^{5}744$	$0.9^{5}756$	$0.9^{5}768$	$0.9^{5}778$

Chi2-distribution

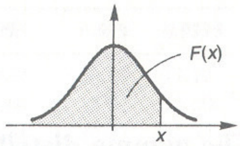
The χ^2-distribution

The table gives x for given values of the distribution function $F(x)$ for a χ^2-distribution with r degrees of freedom.



$F(x)=$	0.0005	0.001	0.005	0.010	0.025	0.05	0.10	0.25	0.50
$r=$ 1	$0.0^6 39$	$0.0^5 16$	$0.0^4 39$	$0.0^3 16$	$0.0^3 98$	0.0039	0.0158	0.1015	0.4549
2	0.0010	0.0020	0.0100	0.0201	0.0506	0.1026	0.2107	0.5754	1.386
3	0.0153	0.0243	0.0717	0.1148	0.2158	0.3518	0.5844	1.213	2.366
4	0.0639	0.0908	0.2070	0.2971	0.4844	0.7107	1.064	1.923	3.357
5	0.158	0.210	0.412	0.554	0.831	1.145	1.610	2.675	4.351
6	0.299	0.381	0.676	0.872	1.237	1.635	2.204	3.455	5.348
7	0.485	0.598	0.989	1.239	1.690	2.167	2.833	4.255	6.346
8	0.710	0.857	1.344	1.646	2.180	2.733	3.490	5.071	7.344
9	0.972	1.153	1.735	2.088	2.700	3.325	4.168	5.899	8.343
10	1.265	1.479	2.156	2.558	3.247	3.940	4.865	6.737	9.342
11	1.587	1.834	2.603	3.053	3.816	4.575	5.578	7.584	10.34
12	1.934	2.214	3.074	3.571	4.404	5.226	6.304	8.438	11.34
13	2.305	2.617	3.565	4.107	5.009	5.892	7.042	9.299	12.34
14	2.697	3.041	4.075	4.660	5.629	6.571	7.790	10.17	13.34
15	3.108	3.483	4.601	5.229	6.262	7.261	8.547	11.04	14.34
16	3.536	3.942	5.142	5.812	6.908	7.962	9.312	11.91	15.34
17	3.980	4.416	5.697	6.408	7.564	8.672	10.09	12.79	16.34
18	4.439	4.905	6.265	7.015	8.231	9.390	10.86	13.68	17.34
19	4.912	5.407	6.844	7.633	8.907	10.12	11.65	14.56	18.34
20	5.398	5.921	7.434	8.260	9.591	10.85	12.44	15.45	19.34
21	5.896	6.447	8.034	8.897	10.28	11.59	13.24	16.34	20.34
22	6.405	6.983	8.643	9.542	10.98	12.34	14.04	17.24	21.34
23	6.924	7.529	9.260	10.20	11.69	13.09	14.85	18.14	22.34
24	7.453	8.085	9.886	10.86	12.40	13.85	15.66	19.04	23.34
25	7.991	8.649	10.52	11.52	13.12	14.61	16.47	19.94	24.34
26	8.538	9.222	11.16	12.20	13.84	15.38	17.29	20.84	25.34
27	9.093	9.803	11.81	12.88	14.57	16.15	18.11	21.75	26.34
28	9.656	10.39	12.46	13.56	15.31	16.93	18.94	22.66	27.34
29	10.23	10.99	13.12	14.26	16.05	17.71	19.77	23.57	28.34
30	10.80	11.59	13.79	14.95	16.79	18.49	20.60	24.48	29.34
34	13.18	14.06	16.50	17.79	19.81	21.66	23.95	28.14	33.34
39	16.27	17.26	20.00	21.43	23.65	25.70	28.20	32.74	38.34
44	19.48	20.58	23.58	25.15	27.58	29.79	32.49	37.36	43.34
49	22.79	23.98	27.25	28.94	31.56	33.93	36.82	42.01	48.34
59	29.64	31.02	34.77	36.70	39.66	42.34	45.58	51.36	58.34
69	36.74	38.30	42.49	44.64	47.92	50.88	54.44	60.76	68.33
79	44.05	45.76	50.38	52.72	56.31	59.52	63.38	70.20	78.33
89	51.52	53.39	58.39	60.93	64.79	68.25	72.39	79.68	88.33
99	59.13	61.14	66.51	69.23	73.36	77.05	81.45	89.18	98.33

Ex. $0.0^3 16 = 0.00016$



The t-distribution

The table gives x for given values of the distribution function $F(x)$ for a t-distribution with r degrees of freedom. For $x<0$ values of $F(x)$ can be obtained from $F(-x)=1-F(x)$.

$F(x)=$	0.75	0.90	0.95	0.975	0.990	0.995	0.9975	0.9995
$r=$ 1	1.0000	3.078	6.314	12.71	31.82	63.66	127.3	636.6
2	0.8165	1.886	2.920	4.303	6.965	9.925	14.09	31.60
3	0.7649	1.638	2.353	3.182	4.541	5.841	7.453	12.92
4	0.7407	1.533	2.132	2.776	3.747	4.604	5.598	8.610
5	0.7267	1.476	2.015	2.571	3.365	4.032	4.773	6.869
6	0.7176	1.440	1.943	2.447	3.143	3.707	4.317	5.959
7	0.7111	1.415	1.895	2.365	2.998	3.499	4.029	5.408
8	0.7064	1.397	1.860	2.306	2.896	3.355	3.832	5.041
9	0.7027	1.383	1.833	2.262	2.821	3.250	3.690	4.781
10	0.6998	1.372	1.812	2.228	2.764	3.169	3.581	4.587
11	0.6974	1.363	1.796	2.201	2.718	3.106	3.497	4.437
12	0.6955	1.356	1.782	2.179	2.681	3.055	3.428	4.318
13	0.6938	1.350	1.771	2.160	2.650	3.012	3.372	4.221
14	0.6924	1.345	1.761	2.145	2.624	2.977	3.326	4.140
15	0.6912	1.341	1.753	2.131	2.602	2.947	3.286	4.073
16	0.6901	1.337	1.746	2.120	2.583	2.921	3.252	4.015
17	0.6892	1.333	1.740	2.110	2.567	2.898	3.222	3.965
18	0.6884	1.330	1.734	2.101	2.552	2.878	3.197	3.922
19	0.6876	1.328	1.729	2.093	2.539	2.861	3.174	3.883
20	0.6870	1.325	1.725	2.086	2.528	2.845	3.153	3.850
21	0.6864	1.323	1.721	2.080	2.518	2.831	3.135	3.819
22	0.6858	1.321	1.717	2.074	2.508	2.819	3.119	3.792
23	0.6853	1.319	1.714	2.069	2.500	2.807	3.104	3.767
24	0.6848	1.318	1.711	2.064	2.492	2.797	3.090	3.745
25	0.6844	1.316	1.708	2.060	2.485	2.787	3.078	3.725
26	0.6840	1.315	1.706	2.056	2.479	2.779	3.069	3.707
27	0.6837	1.314	1.703	2.052	2.473	2.771	3.056	3.690
28	0.6834	1.313	1.701	2.048	2.467	2.763	3.047	3.674
29	0.6830	1.311	1.699	2.045	2.462	2.756	3.038	3.659
30	0.6828	1.310	1.697	2.042	2.457	2.750	3.030	3.646
34	0.6818	1.307	1.691	2.032	2.441	2.728	3.002	3.601
39	0.6808	1.304	1.685	2.023	2.426	2.708	2.976	3.559
44	0.6801	1.301	1.680	2.015	2.414	2.692	2.956	3.526
49	0.6795	1.299	1.677	2.010	2.405	2.680	2.940	3.501
59	0.6787	1.296	1.671	2.001	2.391	2.662	2.916	3.464
69	0.6781	1.294	1.667	1.995	2.382	2.649	2.900	3.438
79	0.6776	1.292	1.664	1.990	2.374	2.640	2.888	3.418
89	0.6773	1.291	1.662	1.987	2.369	2.632	2.879	3.404
99	0.6770	1.290	1.660	1.984	2.365	2.626	2.871	3.392
∞	0.6745	1.282	1.645	1.960	2.326	2.576	2.807	3.291

Table 14-4 Sample Size Code Letters (MIL STD 105E, Table 1)

Lot or Batch Size	Special Inspection Levels S-1	S-2	S-3	S-4	General Inspection Levels I	II	III
2 to 8	A	A	A	A	A	A	B
9 to 15	A	A	A	A	A	B	C
16 to 25	A	A	B	B	B	C	D
26 to 50	A	B	B	C	C	D	E
51 to 90	B	B	C	C	C	E	F
91 to 150	B	B	C	D	D	F	G
151 to 280	B	C	D	E	E	G	H
281 to 500	B	C	D	E	F	H	J
501 to 1200	C	C	E	F	G	J	K
1201 to 3200	C	D	E	G	H	K	L
3201 to 10000	C	D	F	G	J	L	M
10001 to 35000	C	D	F	H	K	M	N
35001 to 150000	D	E	G	J	L	N	P
150001 to 500000	D	E	G	J	M	P	Q
500001 and over	D	E	H	K	N	Q	R

Table 14-6 Master Table for Tightened Inspection—Single Sampling (MIL STD 105E, Table II-B)

Sample Size Code Letter	Sample Size	Acceptable Quality Levels (tightened inspection) 0.010	0.015	0.025	0.040	0.065	0.10	0.15	0.25	0.40	0.65	1.0	1.5	2.5	4.0	6.5	10	15	25	40	65	100	150	250	400	650	1000
		Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re
A	2	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	1 2	2 3	3 4	5 6	8 9	12 13	18 19	27 28
B	3	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	8 9	12 13	18 19	27 28	41 42
C	5	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	8 9	12 13	18 19	27 28	41 42	↑
D	8	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	8 9	12 13	18 19	27 28	41 42	↑	↑
E	13	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	8 9	12 13	18 19	27 28	41 42	↑	↑	↑
F	20	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	8 9	12 13	18 19	↑	↑	↑	↑	↑	↑
G	32	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	8 9	12 13	18 19	↑	↑	↑	↑	↑	↑	↑
H	50	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	8 9	12 13	18 19	↑	↑	↑	↑	↑	↑	↑	↑
J	80	↓	↓	↓	↓	↓	↓	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	8 9	12 13	18 19	↑	↑	↑	↑	↑	↑	↑	↑	↑
K	125	↓	↓	↓	↓	↓	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	8 9	12 13	18 19	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
L	200	↓	↓	↓	↓	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	8 9	12 13	18 19	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
M	315	↓	↓	↓	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	8 9	12 13	18 19	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
N	500	↓	↓	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	8 9	12 11	18 19	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
P	800	↓	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	8 9	12 13	18 19	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
Q	1250	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	8 9	12 13	18 19	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
R	2000	0 1	↑	↓	1 2	2 3	3 4	5 6	8 9	12 13	18 19	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
S	3150			1 2																							

↓ = Use first sampling plan below arrow. If sample size equals, or exceeds, lot or batch size, do 100% inspection.

↑ = Use first sampling plan above arrow.

Ac = Acceptance number.

Re = Rejection number.

Table 14-7 Master Table for Reduced Inspection—Single Sampling (MIL STD 105E, Table II-C)

Sample Size Code Letter	Sample Size	Acceptable Quality Levels (reduced inspection)† 0.010	0.015	0.025	0.040	0.065	0.10	0.15	0.25	0.40	0.65	1.0	1.5	2.5	4.0	6.5	10	15	25	40	65	100	150	250	400	650	1000
		Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re
A	2	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	30 31
B	2	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↑	↓	0 2	1 3	2 4	3 5	5 6	7 8	10 11	14 15	21 22	30 31
C	2	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↑	↓	0 2	1 3	1 4	2 5	3 6	5 8	7 10	10 13	14 17	21 24	
D	3	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↑	↓	0 2	1 3	1 4	2 5	3 6	5 8	7 10	10 13	14 17	21 24	↑	↑
E	5	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↑	↓	0 2	1 3	1 4	2 5	3 6	5 8	7 10	10 13	14 17	21 24	↑	↑	↑
F	8	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↑	↓	0 2	1 3	1 4	2 5	3 6	5 8	7 10	10 13	↑	↑	↑	↑	↑	↑
G	13	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↑	↓	0 2	1 3	1 4	2 5	3 6	5 8	7 10	10 13	↑	↑	↑	↑	↑	↑	↑
H	20	↓	↓	↓	↓	↓	↓	↓	0 1	↑	↓	0 2	1 3	1 4	2 5	3 6	5 8	7 10	10 13	↑	↑	↑	↑	↑	↑	↑	↑
J	32	↓	↓	↓	↓	↓	↓	0 1	↑	↓	0 2	1 3	1 4	2 5	3 6	5 8	7 10	10 13	↑	↑	↑	↑	↑	↑	↑	↑	↑
K	50	↓	↓	↓	↓	↓	0 1	↑	↓	0 2	1 3	1 4	2 5	3 6	5 8	7 10	10 13	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
L	80	↓	↓	↓	↓	0 1	↑	↓	0 2	1 3	1 4	2 5	3 6	5 8	7 10	10 13	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
M	125	↓	↓	↓	0 1	↑	↓	0 2	1 3	1 4	2 5	3 6	5 8	7 10	10 13	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
N	200	↓	↓	0 1	↑	↓	0 2	1 3	1 4	2 5	3 6	5 8	7 10	10 13	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
P	315	↓	0 1	↑	↓	0 2	1 3	1 4	2 5	3 6	5 8	7 10	10 13	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
Q	500	0 1	↑	↓	0 2	1 3	1 4	2 5	3 6	5 8	7 10	10 13	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
R	800	↑	↑	0 2	1 3	1 4	2 5	3 6	5 8	7 10	10 13	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑

↓ = Use first sampling plan below arrow. If sample size equals, or exceeds, lot or batch size, do 100% inspection.

↑ = Use first sampling plan above arrow.

Ac = Acceptance number.

Re = Rejection number.

† = If the acceptance number has been exceeded, but the rejection number has not been reached, accept the lot, but reinstate normal inspection.

Table 14-5 Master Table for Normal Inspection for Single Sampling (MIL STD 105E, Table II-A)

Sample Size Code Letter	Sample Size	Acceptable Quality Levels (normal inspection)																									
		0.010	0.015	0.025	0.040	0.065	0.10	0.15	0.25	0.40	0.65	1.0	1.5	2.5	4.0	6.5	10	15	25	40	65	100	150	250	400	650	1000
		Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re
A	2	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	30 31
B	3	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	30 31	44 45
C	5	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	30 31	44 45	
D	8	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	30 31	44 45	↑	↑
E	13	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	30 31	44 45	↑	↑	↑
F	20	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	↑	↑	↑	↑	↑	↑
G	32	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	↑	↑	↑	↑	↑	↑	↑
H	50	↓	↓	↓	↓	↓	↓	↓	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	↑	↑	↑	↑	↑	↑	↑	↑
J	80	↓	↓	↓	↓	↓	↓	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	↑	↑	↑	↑	↑	↑	↑	↑	↑
K	125	↓	↓	↓	↓	↓	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
L	200	↓	↓	↓	↓	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
M	315	↓	↓	↓	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
N	500	↓	↓	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
P	800	↓	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
Q	1250	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
R	2000	↑	↑	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑

↓ = Use first sampling plan below arrow. If sample size equals, or exceeds, lot or batch size, do 100% inspection.

↑ = Use first sampling plan above arrow.

Ac = Acceptance number.

Re = Rejection number.

Appendix VI Factors for Constructing Variables Control Charts

Observations in Sample, n	Chart for Averages: Factors for Control Limits			Chart for Standard Deviations: Factors for Center Line		Factors for Control Limits				Chart for Ranges: Factors for Center Line		Factors for Control Limits				
	A	A_2	A_3	c_4	$1/c_4$	B_3	B_4	B_5	B_6	d_2	$1/d_2$	d_3	D_1	D_2	D_3	D_4
2	2.121	1.880	2.659	0.7979	1.2533	0	3.267	0	2.606	1.128	0.8865	0.853	0	3.686	0	3.267
3	1.732	1.023	1.954	0.8862	1.1284	0	2.568	0	2.276	1.693	0.5907	0.888	0	4.358	0	2.574
4	1.500	0.729	1.628	0.9213	1.0854	0	2.266	0	2.088	2.059	0.4857	0.880	0	4.698	0	2.282
5	1.342	0.577	1.427	0.9400	1.0638	0	2.089	0	1.964	2.326	0.4299	0.864	0	4.918	0	2.114
6	1.225	0.483	1.287	0.9515	1.0510	0.030	1.970	0.029	1.874	2.534	0.3946	0.848	0	5.078	0	2.004
7	1.134	0.419	1.182	0.9594	1.0423	0.118	1.882	0.113	1.806	2.704	0.3698	0.833	0.204	5.204	0.076	1.924
8	1.061	0.373	1.099	0.9650	1.0363	0.185	1.815	0.179	1.751	2.847	0.3512	0.820	0.388	5.306	0.136	1.864
9	1.000	0.337	1.032	0.9693	1.0317	0.239	1.761	0.232	1.707	2.970	0.3367	0.808	0.547	5.393	0.184	1.816
10	0.949	0.308	0.975	0.9727	1.0281	0.284	1.716	0.276	1.669	3.078	0.3249	0.797	0.687	5.469	0.223	1.777
11	0.905	0.285	0.927	0.9754	1.0252	0.321	1.679	0.313	1.637	3.173	0.3152	0.787	0.811	5.535	0.256	1.744
12	0.866	0.266	0.886	0.9776	1.0229	0.354	1.646	0.346	1.610	3.258	0.3069	0.778	0.922	5.594	0.283	1.717
13	0.832	0.249	0.850	0.9794	1.0210	0.382	1.618	0.374	1.585	3.336	0.2998	0.770	1.025	5.647	0.307	1.693
14	0.802	0.235	0.817	0.9810	1.0194	0.406	1.594	0.399	1.563	3.407	0.2935	0.763	1.118	5.696	0.328	1.672
15	0.775	0.223	0.789	0.9823	1.0180	0.428	1.572	0.421	1.544	3.472	0.2880	0.756	1.203	5.741	0.347	1.653
16	0.750	0.212	0.763	0.9835	1.0168	0.448	1.552	0.440	1.526	3.532	0.2831	0.750	1.282	5.782	0.363	1.637
17	0.728	0.203	0.739	0.9845	1.0157	0.466	1.534	0.458	1.511	3.588	0.2787	0.744	1.356	5.820	0.378	1.622
18	0.707	0.194	0.718	0.9854	1.0148	0.482	1.518	0.475	1.496	3.640	0.2747	0.739	1.424	5.856	0.391	1.608
19	0.688	0.187	0.698	0.9862	1.0140	0.497	1.503	0.490	1.483	3.689	0.2711	0.734	1.487	5.891	0.403	1.597
20	0.671	0.180	0.680	0.9869	1.0133	0.510	1.490	0.504	1.470	3.735	0.2677	0.729	1.549	5.921	0.415	1.585
21	0.655	0.173	0.663	0.9876	1.0126	0.523	1.477	0.516	1.459	3.778	0.2647	0.724	1.605	5.951	0.425	1.575
22	0.640	0.167	0.647	0.9882	1.0119	0.534	1.466	0.528	1.448	3.819	0.2618	0.720	1.659	5.979	0.434	1.566
23	0.626	0.162	0.633	0.9887	1.0114	0.545	1.455	0.539	1.438	3.858	0.2592	0.716	1.710	6.006	0.443	1.557
24	0.612	0.157	0.619	0.9892	1.0109	0.555	1.445	0.549	1.429	3.895	0.2567	0.712	1.759	6.031	0.451	1.548
25	0.600	0.153	0.606	0.9896	1.0105	0.565	1.435	0.559	1.420	3.931	0.2544	0.708	1.806	6.056	0.459	1.541

For $n > 25$.

$$A = \frac{3}{\sqrt{n}} \qquad A_3 = \frac{3}{c_4\sqrt{n}} \qquad c_4 \cong \frac{4(n-1)}{4n-3}$$

$$B_3 = 1 - \frac{3}{c_4\sqrt{2(n-1)}} \qquad B_4 = 1 + \frac{3}{c_4\sqrt{2(n-1)}}$$

$$B_5 = c_4 - \frac{3}{\sqrt{2(n-1)}} \qquad B_6 = c_4 + \frac{3}{\sqrt{2(n-1)}}$$