Exam in mathematical statistics, Statistical Quality Control (MVE-145/MSG-600)

Tuesday 2007-12-18, 8.30-12.30

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Grade limits GU: **G**: 12-21.5, **VG**: 22-30
Grade limits CTH: **3**: 12-17.5, **4**: 18-23.5, **5**: 24-30

Aid problems 1-2: None. Hand-in separately
Aid problems 3-8: One ***handwritten*** A4-page, tables (Beta or similar) and approved calculator (Chalmersgodkänd räknare)

Langugages: You can choose to write in english or in swedish.

Problem 1 (3.5 p)

Solution:
See Montgomery

Problem 2 (4.5p)

Solution:
See Montgomery. Problem 9: Discuss models.

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Hand in part I
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Problem 3 (3p)

SOLUTION

a) Batch size N=7600 and General Inspection Level II gives a code letter L.
    a. For Normal inspection code letter L gives a sample size of 200 and Ac=2, Re=3.
    b. For Tightened inspection, the code letter L gives a sample size of 200 and Ac=1, Re=2.
    c. For Reduced inspection, the code letter L gives a sample size of 80 and Ac=1, Re=3
    d. A comment on switching rules is mandatory.
b) See chapter 14-1.

c) $\begin{cases} n = 200 \\ P(X=0) = \sum_{k=0}^{0} \binom{n}{k} p_U^k (1-p_u)^{n-k} = 0.05 \\ p_u = 1 - 0.05^{\frac{1}{n}} = 1 - 0.05^{\frac{1}{200}} \approx 1.5\% \end{cases}$

Problem 4 (5p)

Solution

a.

The process standard deviation can be estimated by

$$\hat{\sigma} = \frac{\overline{R}}{d_2} = \frac{\frac{9}{25}}{2.326} \approx 0.1548$$

and

$$\hat{\mu} = \frac{\sum_{i=1}^{m} \overline{x}_i}{m} = \frac{662.5}{25} \approx 26.5$$

The natural tolerance limits is then

$USLN = \hat{\mu} + 3\hat{\sigma} = 26.96$

$LSLN = \hat{\mu} - 3\hat{\sigma} = 26.04$

b. control charts

Control limit for the x-chart

$$\begin{cases} UCL = \overline{\overline{x}} + A_2\overline{R} = 26.5 + 0.577\frac{9}{25} \approx 26.70 \\ CL = \overline{\overline{x}} = 26.5 \\ LCL = \overline{\overline{x}} - A_2\overline{R} = 26.5 - 0.577\frac{9}{25} \approx 26.29 \end{cases}$$

Control limits for R-chart

$$\begin{cases} UCL = D_4\overline{R} = 2.114 \cdot \frac{9}{25} = 0.761 \\ CL = \overline{R} = 0.36 \\ LCL = D_3\overline{R} = 0 \end{cases}$$

c. Fraction non-conforming

Use the calculated average and standard deviation from a.

$p = 1 - \left(\Phi\left(25.9 \le X \le 26.9\right)\right) = 1 - \left(\Phi\left(2.58\right) - \Phi\left(-3.88\right)\right) \approx 0.0051$

d. ATS

Probability of an alarm

$$1-P\left(LCL\le\bar{x}\le UCL\right)=$$

$$1-\left(\Phi\left(\frac{26.70-26.4}{\frac{0.1538}{\sqrt{5}}}\right)-\Phi\left(\frac{26.29-26.4}{\frac{0.1538}{\sqrt{5}}}\right)\right)=$$

$$1-\left(\Phi(4.36)-\Phi(-1.60)\right)=1-0.99999350+1-0.945=0.0548$$

$$ARL=\frac{1}{0.0548}\approx 18.24$$

$$ATS=ARL\cdot h=18.24\cdot 0.5=9.12h$$

Problem 5 (4p) Euler buckling

Solution

a. Mean and variance

Since P is a non-linear function of E, I and L I use the equations in 7-7.2 for non-linear combinations

The average:

$$\mu_P=\frac{\pi^2\mu_E\mu_I}{\mu_L^2}=5.07\cdot 10^5\,N \tag{1.1}$$

And the variance becomes:

$$\sigma_P^2\approx\left(\frac{\pi^2\mu_I}{\mu_L^2}\right)^2\sigma_E^2+\left(\frac{\pi^2\mu_E}{\mu_L^2}\right)^2\sigma_I^2+\left(-2\frac{\pi^2\mu_E\mu_I}{\mu_L^3}\right)^2\sigma_L^2=2.5631\cdot 10^8\,N^2 \tag{1.2}$$

b. Reduction of variance.

The variation is to high, so where should we put our (limited) resources? Should we reduce variation in E (buy better steel), decrease variation in L (use better cutting tool) or reduce variation in I (buy better beam)?

$$\begin{aligned}
&\left(\frac{\pi^2\mu_I}{\mu_L^2}\right)^2\sigma_E^2=1.48\cdot 10^8\,N^2\ \text{(Contribution E-module)}\\
&\left(\frac{\pi^2\mu_E}{\mu_L^2}\right)^2\sigma_I^2=1.03\cdot 10^8\,N^2\ \text{(Contribution moment of intertia)}\\
&\left(-2\frac{\pi^2\mu_E\mu_I}{\mu_L^3}\right)^2\sigma_L^2=5.08\cdot 10^6\,N^2\ \text{(Contribution from length)}
\end{aligned} \tag{1.3}$$

The calculation shows that the E-module contributes most to the variation

c. Is the process capable?

We assume that P is approximatively normally distributed. We know that the parameters are normally distriubuted. (A simulation shown in Figure 1 shows that this assumption probably is rather good. This simulation was outside what was needed to do at the exam.)

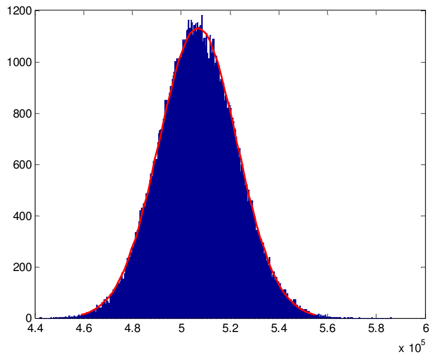


Figure 1 Simulation of 100000 P-values

We calculate the standard deviation to

$$\sigma_P = \sqrt{\sigma_P^2} = 1.59 \cdot 10^4 N \tag{1.4}$$

and we can calculate the capability indices to och

$$\begin{aligned} C_P &= 1.47 \\ C_{pk} &= 0.77 \end{aligned} \tag{1.5}$$

and comes to the conclusion that the variation is OK, but the process needs to be centered.

Problem6 (3p)

SOLUTION

$P(\text{Alarm within 3 samples}) =$

$= 1 - P(\text{No alarm within 3 samples}) =$

$1 - P(\text{Sample 1: no alarm AND Sample 2: no alarm AND Sample 3: no alarm}) =$

$= 1 - \left(P(\text{No alarm})\right)^3$

$P(\text{No alarm}) = P\left(\text{average within control limits}\right) = P\left(LCL \le \overline{x} \le UCL\right) =$

$= P(\overline{x} \le UCL) - P(\overline{x} \le LCL) =$

$$= \Phi\left(\frac{104-98}{8/\sqrt{5}}\right) - \Phi\left(\frac{96-98}{8/\sqrt{5}}\right) = \Phi(1.68) - \Phi(-0.56) =$$

$= 0.9532 - 0.2881 = 0.6652$

$P\left(\text{alarm within 3 samples}\right) = 1 - 0.6652^3 = 0.7057$

Uppgift 7 (4p)

Solution

a. Process standard deviation

The process standard deviation can be estimated from the moving range:

$$\overline{MR} = \frac{442}{19} = 23.2632$$

$$\hat{\sigma} = \frac{\overline{MR}}{d_2} = \frac{23.2632}{1.128} \approx 20.62$$

b. EWMA-diagram

The control limits for the EWMA-diagram becomes:

$$UCL = \mu_0 + L\sigma\sqrt{\frac{\lambda}{2-\lambda}\left[1-(1-\lambda)^{2i}\right]}$$

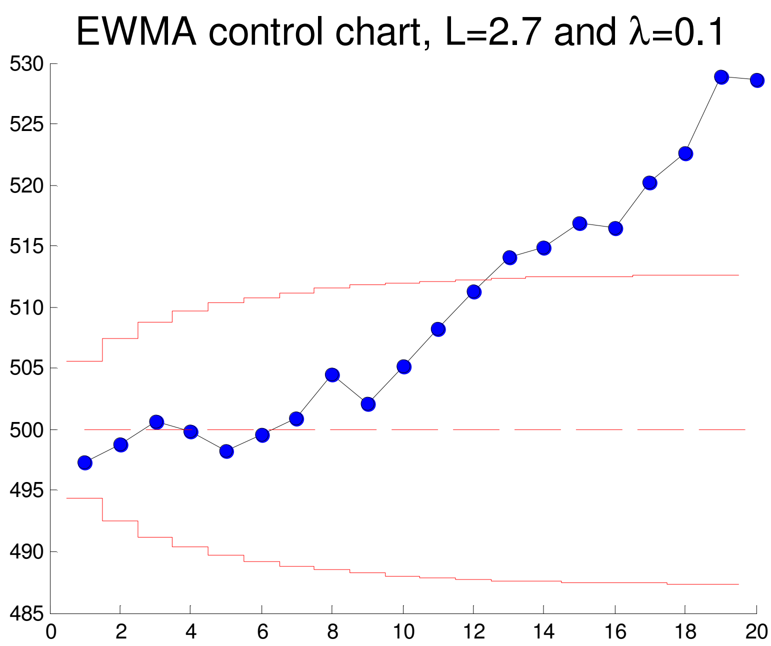
$$CL = \mu_0$$

$$LCL = \mu_0 - L\sigma\sqrt{\frac{\lambda}{2-\lambda}\left[1-(1-\lambda)^{2i}\right]}$$

We choose to calculate with $\lambda = 0.1$ and L=2.7.

$z_i = \lambda x_i + (1-\lambda) z_{i-1},\ \ z_0 = \mu_0$

i	z_i	UCL	CL	LCL
1	497	506	500	494
2	499	507	500	493
3	501	509	500	491
4	500	510	500	490
5	498	510	500	490
6	500	511	500	489
7	501	511	500	489
8	504	512	500	488
9	502	512	500	488
10	505	512	500	488
11	508	512	500	488
12	511	512	500	488
13	514	512	500	488
14	515	512	500	488
15	517	513	500	487
16	517	513	500	487
17	520	513	500	487
18	523	513	500	487
19	529	513	500	487
20	529	513	500	487



Out-of control signal at sample i=13! (The table shows rounded values.)

Uppgift 8 (3p)

Solution

Answer: $p = \Phi\left(-3C_{pk}\right) + \Phi\left(-3\left(2C_p - C_{pk}\right)\right)$

Derivaiton

$$p = 1 - P\left(LSL \le X \le USL\right) = 1 - P\left(\frac{LSL-\mu}{\sigma} \le \underbrace{\frac{x-\mu}{\sigma}}_{N(0,1)} \le \frac{USL-\mu}{\sigma}\right) =$$

$$= \Phi\left(\frac{LSL-\mu}{\sigma}\right) + \left(1 - \Phi\left(\frac{USL-\mu}{\sigma}\right)\right)$$

If $\mu \le \dfrac{USL+LSL}{2}$ the first part can be written as

$$\Phi\left(\frac{LSL-\mu}{\sigma}\right) = \Phi\left(-\frac{\mu-LSL}{\sigma}\right) = \Phi\left(-3\left(\frac{\mu-LSL}{3\sigma}\right)\right) = \Phi\left(-3C_{pk}\right)$$

and the second part can be written as

$$1-\Phi\left(\frac{USL-\mu}{\sigma}\right) = \Phi\left(\frac{\mu-USL}{\sigma}\right) = \Phi\left(\frac{\mu-LSL}{\sigma} + \frac{\mu-USL}{\sigma} - \frac{\mu-LSL}{\sigma}\right) =$$

$$\Phi\left(\frac{\mu-LSL}{\sigma} - \frac{USL-LSL}{\sigma}\right) = \Phi\left(3\underbrace{\left(\frac{\mu-LSL}{3\sigma}\right)}_{C_{pk}} - 6\underbrace{\left(\frac{USL-LSL}{6\sigma}\right)}_{C_p}\right) = \Phi\left(-3\left(2C_p - C_{pk}\right)\right)$$

Symmetry gives the same result for $\mu > \dfrac{USL+LSL}{2}$.

The fraction outside tolerances is then:

$$p = \Phi\left(-3C_{pk}\right) + \Phi\left(-3\left(2C_p - C_{pk}\right)\right) = \Phi\left(-3\cdot 1.33\right) + \Phi\left(-3\left(2\cdot 1.67 - 1.33\right)\right) =$$
$$\Phi\left(-3.99\right) + \Phi\left(-6.03\right) \approx 1 - 0.99997 + 0 = 3\cdot 10^{-5}$$