Lesson 2

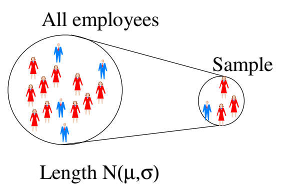
2007-11-07

Chaptre 3

Projects and Hand-ins

- Project 1
  - In: latest 21 november
- Project 2
  - In: latest 5 december
- Laboration Measurement systems analysis
  - In: latest 5 december
- Project 1-2 are voluntary. Laboration is obligatory.
- Give up to 1 bonus point each to the exam 18 dec.
- Project 1-2 shall be done individually. Laboration should be done in groups (2-4 persons).
- Projects and laboration is available on the homepage. Slide caliper can be borrowed from the teacher.

Hypothesis testing



$$\begin{cases} \{174 \quad 166 \quad 171 \quad 174 \quad 175\} \\ \bar{x} = 172.0 \\ s = 3.67 \end{cases}$$

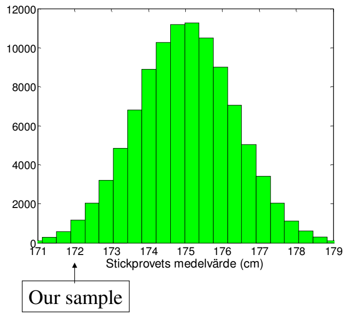
Claim: The average length of all employes is 175cm

The sample average is = 172.0cm.

Är the claim true or not?

Hypotesis length

- Suppose that the length of the employees N(175,3) cm.
- Generereate 100000 random samples of size 5.
- How unusual is our



Hypotesis length

- There are 1271 sample averages less that 172.0 cm.
- There are 98729 sample averages greater than 172.0
- Our sample is rather unusual **if** the length realley is N(175,3)-distributed.
- If the population mean length were lower than 175cm then should our sample not be so unusual.

Testing hypothesis

- *Hypotesis* – an assumption about the population (about a
- *Testing hypotesis* – does the the sample data support the hypothesis or not?

$$\begin{cases} H_0 & \text{Zero hypothesis} \\ H_1 & \text{Alternative hypothesis} \end{cases}$$

Hypotesis examples

- Suppose that the average weight of a bandage is 5g.

$$\begin{cases} H_0 & \mu = 5 \\ H_1 & \mu \neq 5 \end{cases}$$

- Toss a coin 10 times. The fraction ”heads” is p.

$$\begin{cases} H_0 & p = 0.5 \\ H_1 & p > 0.5 \end{cases}$$

Hypotesis test

1. Write down zero hypothesis H_0.
2. Write down alternative hypothesis H_1.
3. Decide a significance level α.
4. Decide a test function (test statistica).
5. Take a random sample and calculate the values (mean, standard deviation et.c.) that are needed in
6. Calculate a value on the test
7. Decide the critical area and determine if the zero hypothesis can be rejected or not.

zero hypothesis and alternative

- Zero hypothesis H_0.
  - H_0 is a statement of the population.
- Alternative hypothesis H_1.
  - Complement to H_0.
  - One sided

    $$H_1 : \quad \mu < 175cm$$
  - Double sided

    $$H_1 : \quad \mu \neq 175cm$$

Significance level

- Significance
  - shown statistically
  - significant different (in swedish)
- Signifikance level α
  - The risk of rejecting H_0 if H_0 is true.
  - **Exemple:** *If the zero hypothesis is true and α=0.05 then on average the zero hypothesis will wrongly be rejected on average every 20 test.*

Test function

- The test function is a function fo the sample, and hence is random.
- Exampel normally distributed values with known variance σ^2:

$$Z_0 = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} \sim N(0,1)$$

Sample and test function

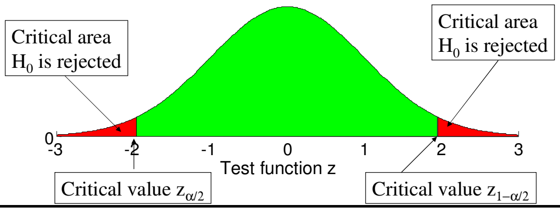
- Take a sample
- Calculate the value of the test function

$$\begin{cases} \mu = 175 \\ \sigma = 3 \\ \bar{x} = 172.0 \\ n = 5 \end{cases}$$

$$z_0 = \frac{172 - 175}{\frac{3}{\sqrt{5}}} \approx -2.24$$

Critical area

- The critical area contains the values where H_0 is rejected.
- The limit of the critical area is called critical value. Is decided byt the significance level α.



Errors when testing hypothesis

	Keep H_0.	Reject H_0.
H_0 true	**OK**	**α-risk**
H_0 false	**β-risk**	**OK**

α = P(Type I error) = P(reject H_0 | H_0 is true)
β = P(Type II error) = P(keep H_0 | H_0 is false)

Error when testing hypothesis

α is the *signifikance level* of the test.
1- β is the *power* of the test.

α is called *producer risk*.
β is called *consumer risk*.

P-value och significance level

"H_0 was rejected on the level 0.05."

Was it near the critical value or was it good marginal?

P-value is the lowest significance level for which H_0 will be rejected.

Exemple: p-value for the length 172 cm is 0.025.

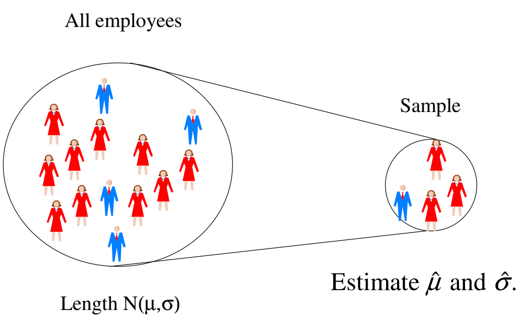
H_0 is not automatically true only because it has not been rejected!

$$\begin{cases} H_0: \text{ Smoking is not dangerous.} \\ H_1: \text{ Smoking is dangerous.} \end{cases}$$

$$\begin{cases} H_0: \text{ There is no life but on earth.} \\ H_1: \text{ There is life outside earth.} \end{cases}$$

The fact that we have not found life on other planets is no eveidence for that there is no life on any other planets.
On the other hand, if ET is found then we can reject H_0

Employee length



Estimate $\hat{\mu}$ and $\hat{\sigma}$.

Estimation of normal distribution parameters.

- Normalfördelningens parameterar kan nu skattas med:

$$\begin{cases} \hat{\mu} = \bar{x} \\ \hat{\sigma}^2 = s^2 \\ \hat{\sigma} \neq s \end{cases}$$

Exemple length

- A sample of size 5 was taken.

$$\{176 \quad 169 \quad 174 \quad 174 \quad 175\}$$
$$\bar{x} = 173.6$$
$$s = 2.70$$

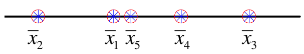
- The parameters can now be estimated to

$$\hat{\mu} = \bar{x} = 173.6$$
$$\hat{\sigma}^2 = s^2 = 7.3$$

Uncertainty

- How sure can we be that our estimation is good enough?
- The sample averages varies from sample to sample.

$$\begin{cases} \bar{x}_1 = 173.6 \\ \bar{x}_2 = 171.8 \\ \bar{x}_3 = 176.8 \\ \bar{x}_4 = 175.2 \\ \bar{x}_5 = 174.0 \end{cases}$$



Confidence interval

- A confidence interval [L, H] is a random interval that fulfills

$$P(L \le \mu \le H) = 1-\alpha$$

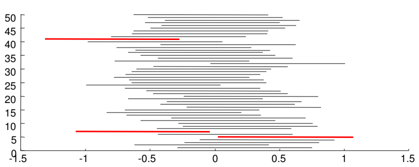
- Eg. 95% confidence interval for μ is an interval that with 95% probabilty contains the true value of μ.
- The confidence interval decreases when the sample size increases.

Confidence interval

- *Confidence level*: The probability that the true value will be contained in the interval. is denoted sometime as (1–α).
- Confidence intervals:
  - Double sided
  - Symmetrical
  - One sided
- Exemple of denotations: $\begin{cases} \mu \in [10,18] & (95\%) \\ \mu = 14 \pm 4 & (95\%) \end{cases}$

Exemple 90% confidence interval

- 50 random N(0,1) samples of size 10.
- 3 of 50 intervals does not contain 0.



Confidence intervals for normal distribution

- We want to find a confidence interval for the parameter μ for a N(μ,σ)–distributed variable.

$$P(L \le \mu \le H) = 1 - \alpha$$

- 2 formulas depending on if
  - σ is known.
  - σ is not known.

Average σ known

- Let X be a N(μ,σ)–distributed stochastic variable.
- Take a sample of size n.
- The sample average is then: $\bar{X} \sim N(\mu, \frac{\sigma}{\sqrt{n}})$
- and the normalized value is

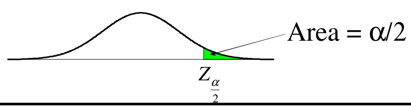
$$Z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} \sim N(0,1)$$

Confidence interval for normal distribution, known variance

- Suppose that a normally distributed N(μ,σ) sample of size n. A (1–α) confidence interval is then:

$$\bar{x} - z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} \le \mu \le \bar{x} + z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$$

- $Z_{\alpha/2}$ is defined by 100(1- α)% percentile of N(0,1).



Sample average unknown variance

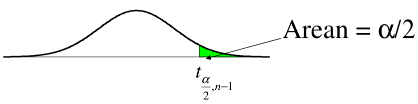
- If σ is unknown then the t-distribution should be used instead of the z-distribution.

$$T = \frac{\overline{X} - \mu}{\frac{s}{\sqrt{n}}} \sim t(\upsilon)$$

- t-distribution si symmetrical and looks like the normal distribution when n is big.
- The parametern ν=n-1denotes the degrees of freedom.

Confidence interval unknown variance

$$\overline{x} - t_{\frac{\alpha}{2},n-1}\frac{s}{\sqrt{n}} \le \mu \le \overline{x} + t_{\frac{\alpha}{2},n-1}\frac{s}{\sqrt{n}}$$



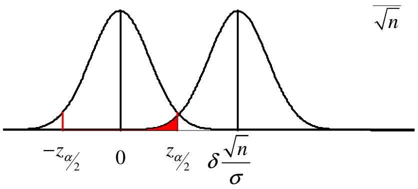
Sample size and Type II -error

How good is the test? $\begin{cases} H_0: & \mu = \mu_0 \\ H_1: & \mu \neq \mu_0 \end{cases}$

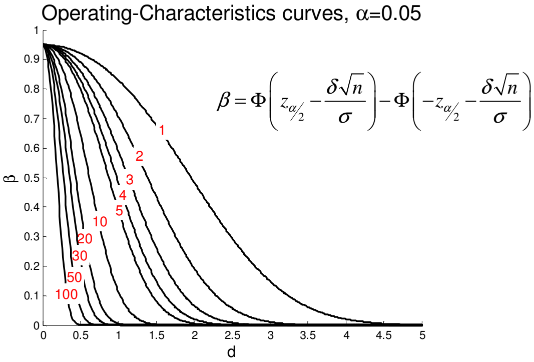
$$Z_0 \sim N(0,1) \text{ under } H_0$$

$$Z_0 \sim N(\frac{\delta\sqrt{n}}{\sigma},1) \text{ under } H_1$$

$$Z_0 = \frac{\overline{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}}$$



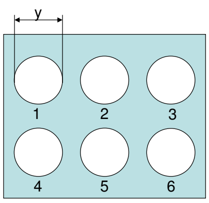
Operating-Characteristics curves, α=0.05



ANOVA
Analysis of variance

- Analysis of 2 samples: chapter 3.4
- Is used in measurement systems analysis in chapter 7
- Comparison of many averages.

Exemple: Mold tool with 6 cavities.



	Cavity number						
Cast nr.	**1**	**2**	**3**	**4**	**5**	**6**	**Average**
1	25.77	25.79	25.81	25.83	25.84	25.86	**25.817**
2	25.77	25.79	25.80	25.83	25.85	25.87	**25.818**
3	25.77	25.78	25.81	25.82	25.86	25.88	**25.820**
4	25.77	25.79	25.82	25.82	25.82	25.87	**25.815**
5	25.78	25.78	25.81	25.82	25.83	25.87	**25.815**
6	25.80	25.79	25.81	25.83	25.84	25.85	**25.820**
7	25.76	25.79	25.81	25.83	25.84	25.87	**25.817**
Average	**25.774**	**25.787**	**25.810**	**25.826**	**25.840**	**25.867**	**25.817**

Is there a real difference between the cavities or is the variation due to chance only?

Mold tool.

Model: $y_{ij} = \mu + \tau_i + \varepsilon_{ij}, \begin{cases} i = 1 \ldots 6 \\ j = 1 \ldots 7 \end{cases}$

$$\sum_{i=1}^{6} \tau_i = 0$$

Suppose $y_{ij} \sim N(\mu + \tau_i, \sigma)$

Test hypotes:

$H_0 : \tau_1 = \tau_2 = \ldots \tau_6 = 0$

to

$H_1 : \tau_i \neq 0$ for at least some i

Mold tool.

We will use variation analysis (ANOVA).
The idea is to divide the variation into two parts, one that comes from the difference between the cavities and another that come from the variation within the cavities.

Source of variation	Sum of squares	Degrees of freedom	Mean Square	F_0
Between treatments	$SS_{Treatments} = n \sum_{i=1}^{n} (\bar{y}_{i.} - \bar{y}_{..})^2$	$a-1$	$MS_{Treatments}$	$F_0 = \frac{MS_{Treatments}}{MS_E}$
Error (within treatments)	$SS_E = SS_T - SS_E$	N-a	MS_E	
Total	$SS_T = \sum_{i=1}^{a} \sum_{j=1}^{n} (y_{ij} - \bar{y}_{..})^2$	N-1		

$$y_{i.} = \sum_{j=1}^{n} y_{ij} \quad y_{..} = \sum_{i=1}^{a} \sum_{j=1}^{n} y_{ij} \quad N = a \cdot n$$

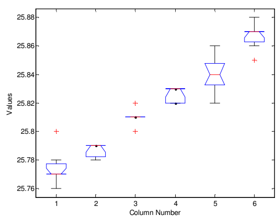
$$\bar{y}_{i.} = y_{i.}/n \quad \bar{y}_{..} = y_{..}/N$$

Molding tool.

ANOVA Table

Source	SS	df	MS	F	Prob>F
Columns	0.04118	5	0.00824	97.91	0
Error	0.00303	36	0.00008		
Total	0.04421	41			

Matlab: anova1(X)



Molding tool, the residuals

Study the residual

$e_{ij} = y_{ij} - \bar{y}_{i.}$

The assumption seems OK!