Lektion 3

2007-11-14

Chapter 5

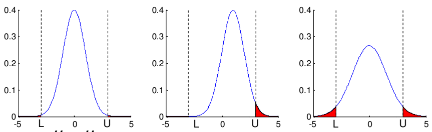
Control Charts for Variables

Sources of variation

- *chance, random causes*
  - Random variation
  - White noise. Background variation
  - In control, stable process
- *Systematic, assignable causes*
  - There is a reason
  - Out of control
  - Process not stable
- The purpose of SPC is to detect and eliminate systematic sources of variation.

Control of variables

- Measurable characteristics of a product.
- Control **both** average and spread!

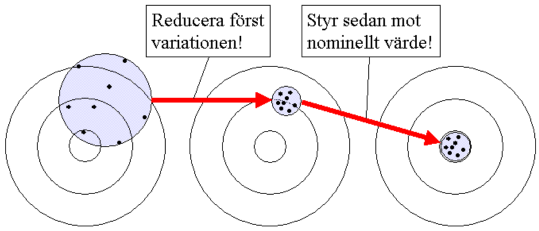


$\mu = \mu_0$ $\quad$ $\mu > \mu_0$ $\quad$ $\mu = \mu_0$

$\sigma = \sigma_0$ $\quad$ $\sigma = \sigma_0$ $\quad$ $\sigma > \sigma_0$

Control charts

- Allways two diagrams



Control limits

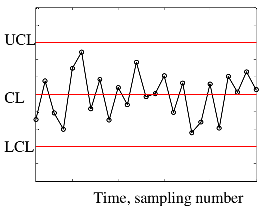
μ and σ are known.

$$X \sim N(\mu, \sigma)$$

$$\bar{X} \sim N(\mu, \frac{\sigma}{\sqrt{n}})$$

$$UCL = \mu + z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

$$LCL = \mu - z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$



$$z_{\alpha/2} = \begin{cases} 3 & \text{Popular limits} \\ 3.09 & \text{Probability limits} \end{cases}$$

We do neither know μ nor σ!

- X-R-diagram
  - No need of computers
  - R more intuitive
  - Vaste of information
- X-S-diagram
  - Computer (calculator) is needed.
  - More efficient.
  - Standard deviation not very easy to understand.

X-R-diagram

m sampling groups

n size of sampling group

$$\hat{\mu} = \overline{\overline{x}} = \frac{\overline{x}_1 + \overline{x}_2 + \ldots + \overline{x}_m}{m}$$

$$\hat{\sigma} = \frac{\overline{R}}{d_2}$$

$$3\frac{\hat{\sigma}}{\sqrt{n}} = \frac{3}{d_2\sqrt{n}}\overline{R} = A_2\overline{R}$$

$$\left[\begin{array}{l} \overline{x}_i = \dfrac{x_{i,1} + x_{i,2} + \ldots + x_{i,n}}{n} \\ \overline{R} = \dfrac{R_1 + R_2 + \ldots + R_m}{m} \\ R_i = \max(x_{ij}) - \min(x_{ij}),\ j = 1, \ldots, n \end{array}\right]$$

A_2 and d_2 are found in appendix page 725

Control limits x-R

Control limits for $\overline{x}$-diagram

$$UCL = \hat{\mu} + 3\frac{\hat{\sigma}}{\sqrt{n}} = \overline{\overline{x}} + A_2\overline{R}$$

$$CL = \hat{\mu} = \overline{\overline{x}}$$

$$LCL = \hat{\mu} - 3\frac{\hat{\sigma}}{\sqrt{n}} = \overline{\overline{x}} - A_2\overline{R}$$

Control limits for R-diagram

$$UCL = \overline{R} + 3\hat{\sigma}_R = D_4\overline{R}$$

$$CL = \overline{R}$$

$$LCL = \overline{R} - 3\hat{\sigma}_R = D_3\overline{R}$$

A_2, D_3 and D_4 are found in appendix page 725

Exempel:

En fabrik tillverkar stålfjädrar med en önskad styvhet på 75 N/mm. Man har en stabil produktion och önskar nu att föra in statistisk processtyrning som en metod för att säkerställa stabiliteten i framtiden. Provtagning sker redan med regelbundna intervall och man tar ut 4 fjädrar i varje prov för styvhetsanalys. Efter att 25 provgrupper har tagits ut så påbörjar man analysen. Det första man undersöker är om fjäderstyvheten är normalfördelad. Data är sammanställda i tabell 5.1.

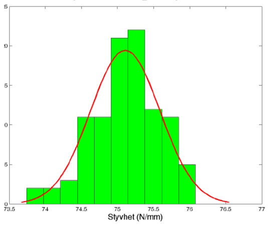
För att avgöra om data är tillräckligt normalfördelade så tillverkas ett histogram över provtagningsdata. I figur 5.6 ser man att det finns inga direkta bevis för att data inte skulle vara normalfördelade. Vi kan alltså gå vidare med metoder för normalfördelade data.

Steel springs with stiffness 75N/mm. 25 sample groups, each of size 4.
Figure 5.6 shows no evidence against normality.

Provgrupp i	$x_{i,1}$	$x_{i,2}$	$x_{i,3}$	$x_{i,4}$	$\bar{x}_i$	R_i	s_i
1	74.92	75.41	75.50	75.16	75.25	0.58	0.26
2	74.70	74.49	75.58	75.37	75.03	1.09	0.52
3	75.34	75.76	75.15	75.43	75.42	0.61	0.25
4	75.29	74.98	75.24	75.40	75.23	0.42	0.18
5	75.83	75.14	74.96	75.53	75.37	0.87	0.39
6	74.86	75.58	75.25	75.27	75.24	0.72	0.29
7	74.97	73.74	74.61	75.23	74.64	1.49	0.65
8	74.73	74.71	74.44	75.31	74.80	0.87	0.37
9	74.05	74.04	74.96	75.50	74.64	1.46	0.72
10	75.57	75.15	74.71	75.04	75.12	0.86	0.35
11	75.87	74.67	74.98	74.81	75.08	1.20	0.54
12	75.32	74.56	75.19	74.26	74.83	1.06	0.51
13	74.99	74.67	75.31	74.47	74.86	0.84	0.37
14	74.51	74.62	74.38	75.11	74.66	0.73	0.32
15	74.71	74.54	75.06	75.78	75.02	1.24	0.55
16	75.10	75.69	74.89	75.64	75.33	0.80	0.40
17	74.80	76.08	74.45	75.30	75.16	1.63	0.71
18	74.71	74.99	75.12	74.73	74.89	0.41	0.20
19	74.99	75.37	75.39	75.69	75.36	0.70	0.29
20	75.84	74.92	75.63	73.79	75.05	2.05	0.92
21	75.56	75.80	75.93	75.16	75.61	0.77	0.34
22	75.87	75.76	74.93	75.24	75.45	0.94	0.44
23	75.04	74.51	75.37	75.02	74.98	0.86	0.36
24	74.97	75.72	75.18	75.05	75.23	0.75	0.34
25	75.85	75.21	75.45	75.20	75.43	0.65	0.30

Tabell 5.1: Data från de 25 första provgrupperna

Histogram of spring stiffness



Figur 5.6: Histogram över provtagningsdata.

$$\begin{cases} \bar{\bar{x}} = 75.1065 \\ \bar{R} = 0.9440 \end{cases}$$

Spring stiffness –ex.

Control limits for $\bar{x}$-diagram

$UCL = \bar{\bar{x}} + A_2\bar{R} = 75.11 + 0.729 \cdot 0.944 = 75.79$

$CL = \bar{\bar{x}} = 75.11$

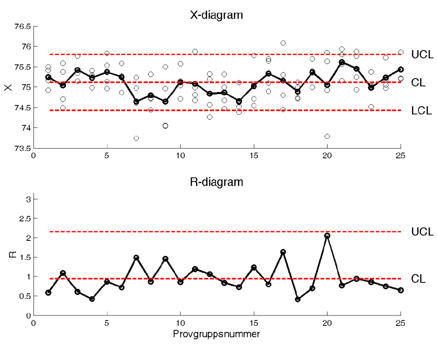
$LCL = \bar{\bar{x}} - A_2\bar{R} = 75.11 - 0.729 \cdot 0.944 = 74.42$

Control limits for R-diagram

$UCL = D_4\bar{R} = 2.282 \cdot 0.944 = 2.15$

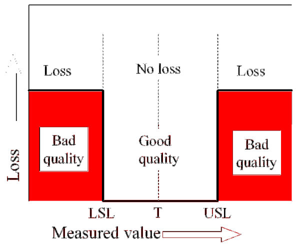
$CL = \bar{R} = 0.944$

$LCL = \bar{R} - 3\hat{\sigma}_R = D_3\bar{R} = 0 \cdot 0.944 = 0$



Figur 5.7: Styrdiagram över fjäderstyvhet.

Is the spring process *capable*?



Toleransgränser

$k = 75 \pm 2\, N/mm$

$$\begin{cases} USL = 77\, N/mm \\ T = 75\, N/mm \\ LSL = 73\, N/mm \end{cases}$$

The spread of spring stiffness

The standard deviation is estimated by:

$$\hat{\sigma} = \frac{\overline{R}}{d_2} = \frac{0.944}{2.059} = 0.46\, N/mm$$

The percentage defective is estimated by:

$$\begin{aligned} \text{p} &= P(x < 73) + P(x > 77) \\ &= \Phi\left(\frac{73 - 75.11}{0.46}\right) + 1 - \Phi\left(\frac{77 - 75.11}{0.46}\right) \\ &= \Phi(-4.59) + 1 - \Phi(4.11) \\ &= 2.21 \cdot 10^{-5} \end{aligned}$$

Process capability

Process Capability Ratio:

$$C_p = \frac{USL - LSL}{6\sigma}$$

$$\hat{C}_p = \frac{USL - LSL}{6\hat{\sigma}} = \frac{77 - 73}{6 \cdot 0.46} = 1.48$$

Process "natural" tolerance limits:

$UNTL = \mu + 3\sigma$

$LNTL = \mu - 3\sigma$

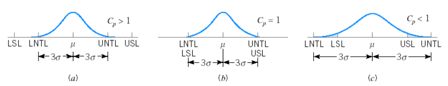


Figure 5-3 Process fallout and the process capability ratio C_p

(More about process capability in ch. 7)

Control limits, tolerance limits and natural tolerance limits

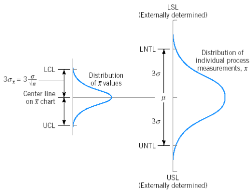
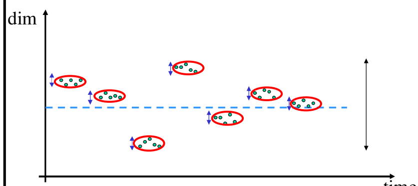


Figure 5-6 Relationship of natural tolerance limits, control limits, and specification limits.

Obs!
There is no connection between control limits (natural tolerance limits) and tolerance limits!

Rational sampling groups

$\bar{x}$ – diagram controls average (Between group variation)
R – diagram controls the variation within the group. (Within group variation)



Estimate σ from within group variation!

A comment on R-method

- R=max – min
  - Easy, but inefficient for large sample group sizes!

n	Relative efficiency
2	1.000
3	0.992
4	0.975
5	0.955
6	0.930
10	0.850

Not so good to detect small shifts in process average.

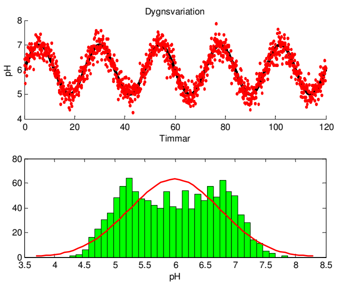
R- and other limits

- Probability limits
  - α=0.002 → 99.9% och 0.01%
  - Välj k=$Z_{\alpha/2}$=3.09 istället för 3 (medevärdediag.)
  - Välj $D_{0.001}\sigma$ och $D_{0.999}\sigma$ som R-gränser.
- Standard values
  - Warning! Do you really know you proces so well?

Interpretation of diagrams

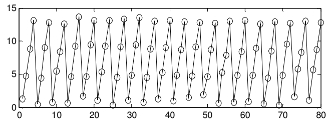
- Cyclic patterns
- Mixes
- Change in process average
- Trends
- Stratification (to small withingroup variation)

Ex: 24 hour variation

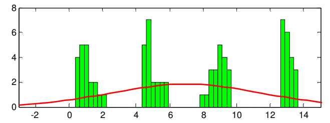


Suggestion: Use a process model that incoorporates the daily variation.

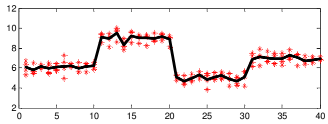
Ex: Cavity variation



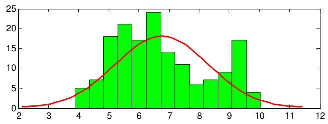
Suggestion: Study every cavity individually.



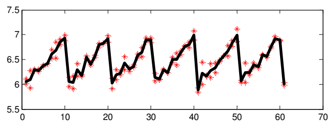
Ex: Variation in raw material.



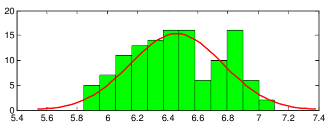
Suggestion: Try to decrease the variation, alternatively to make the product more robust.



Ex: Worn tool.



Suggestion: Special SPC and capability.

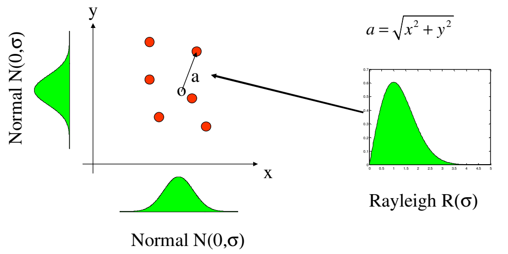


Truncated distribution. Under estimated capability

X-R-diagram and not normal data

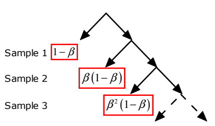
- Avereage diagram rather stable!
  - Central limit theorem (thank!)
  - If n>4 then it is normally OK. (depends on process)
- The spread diagram is sensitive!
  - Unsymmetrical

Ex. Distance to hole center

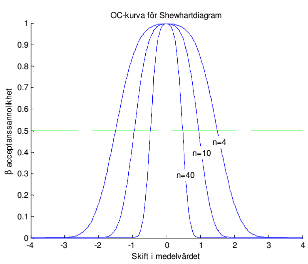


OC-curve and ARL

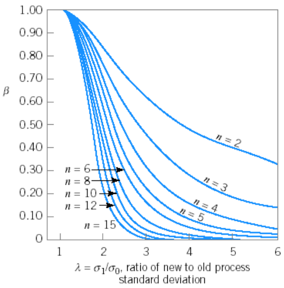
$$\beta = P(LCL \le \bar{x} \le UCL \mid \mu = \mu_0 + k\sigma)$$
$$= \Phi\left(L - k\sqrt{n}\right) - \Phi\left(-L - k\sqrt{n}\right)$$



$$ARL = \sum_{r=1}^{\infty} r\beta^{r-1}(1-\beta) = \frac{1}{1-\beta}$$



OC-kurva för R



R-diagram not effective for small changes in standard deviation.

Control charts for x and s

- Estimate σ with sample standard deviation.
- Computational aid is needed.
- More effective than R for large sample sizes.
- Variable sample sizes possible.

The Diagram

$\bar{x}$-diagram

$$\begin{cases} UCL = \bar{\bar{x}} + 3\dfrac{\bar{s}}{c_4\sqrt{n}} = \bar{\bar{x}} + A_3\bar{s} \\ CL = \bar{\bar{x}} \\ LCL = \bar{\bar{x}} - 3\dfrac{\bar{s}}{c_4\sqrt{n}} = \bar{\bar{x}} - A_3\bar{s} \end{cases}$$

s-diagram

$$\begin{cases} UCL = \bar{s} + 3\dfrac{\bar{s}}{c_4}\sqrt{1-c_4^2} = B_4\bar{s} \\ CL = \bar{s} \\ LCL = \bar{s} - 3\dfrac{\bar{s}}{c_4}\sqrt{1-c_4^2} = B_3\bar{s} \end{cases}$$

$$\bar{x}_i = \frac{x_{i,1} + x_{i,2} + \ldots + x_{i,n}}{n}$$

$$\bar{\bar{x}} = \frac{\bar{x}_1 + \bar{x}_2 + \ldots + \bar{x}_m}{m}$$

$$s_i = \sqrt{\frac{1}{n-1}\sum_{j=1}^{n}(x_{i,j} - \bar{x}_i)^2}$$

$$\bar{s} = \frac{s_1 + s_2 + \ldots + s_m}{m}$$

$$c_4 = \left(\frac{2}{n-1}\right)^{1/2} \frac{\Gamma\left(\frac{n}{2}\right)}{\Gamma\left(\frac{n-1}{2}\right)}$$

See page 725 for constants c_4, B_3, B_4, A_3

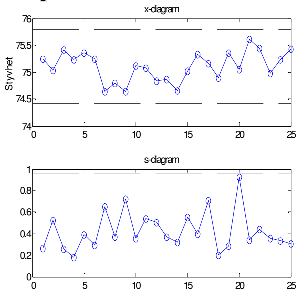
x-s diagram for spring stiffness-example

$\overline{x}$-diagram

$$\begin{cases} UCL = \overline{\overline{x}} + A_3\overline{s} = 75.11 + 1.628 \cdot 0.423 = 75.79 \\ CL = \overline{\overline{x}} = 75.11 \\ LCL = \overline{\overline{x}} - A_3\overline{s} = 74.42 \end{cases}$$

s-diagram

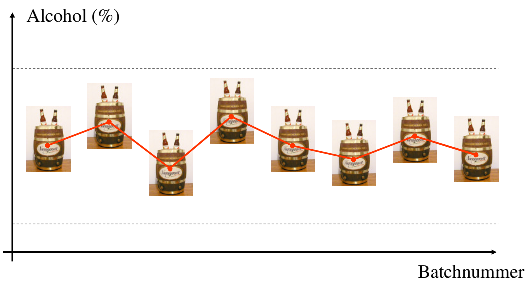
$$\begin{cases} UCL = B_4\overline{s} = 2.266 \cdot 0.423 = 0.96 \\ CL = \overline{s} = 0.43 \\ LCL = B_3\overline{s} = 0 \cdot 0.423 = 0 \end{cases}$$



Shewhart when n=1

- Automtized measurement. No sample groups.
- Very low production speed.
- Bulk production. Only measurement error present.
- Many measurements on the same product.

Variation between barrells



Design of diagram

$\bar{x}$-diagram

$$\begin{cases} UCL = \bar{x} + 3\dfrac{\overline{MR}}{d_2} \\ CL = \bar{x} \\ LCL = \bar{x} - 3\dfrac{\overline{MR}}{d_2} \end{cases}$$

$$MR_i = |x_i - x_{i-1}|$$

$$\overline{MR} = \frac{MR_1 + MR_2 + \ldots + MR_m}{m}$$

MR-diagram

$$\begin{cases} UCL = D_4\overline{MR} \\ CL = \overline{MR} \\ LCL = 0 \end{cases}$$

Exempel MR

Table 5-6 Viscosity of Aircraft Primer Paint

Batch Number	Viscosity x	Moving Range MR
1	34.05	
2	34.40	0.35
3	33.59	0.81
4	35.96	2.37
5	34.70	1.26
6	33.51	1.19
7	33.79	0.28
8	34.04	0.25
9	34.52	0.48
10	33.75	0.77
11	33.27	0.48
12	33.71	0.44
13	34.03	0.32
14	34.58	0.55
15	34.02	0.56
16	33.97	0.05
17	34.05	0.08
18	34.04	0.01
19	33.73	0.31
20	34.05	0.32
	$\bar{x} = 34.088$	$\overline{MR} = 0.5726$

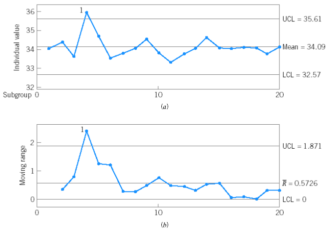


Figure 5-19 Control charts for (a) the moving range and for (b) individual observations on viscosity.

Problem with Moving Range

- Very sensitive for not normal data
- ARL large for small shifts.
- Alternatives CUMSUM or EWMA

- Montgomery warns!