

Lektion 4

2007-11-21

Chapter 6

Attribute control

Sources of variation

- Chance causes (*Slumpmässiga källor*)
 - Random variation
 - Background noise
 - Statistical control, stable process
- Assignable causes (*Systematiska källor*)
 - There is a cause
 - Out of control
 - Not stable
- The purpose of SPC is to detect and eliminate systematic (assignable) sources of variation!

Attribute

- Yes/No – data.
 - Number of defect units in a batch.
 - Binomial distribution
 - p-diagram
 - np-diagram
- Countable data
 - Number of defects (a product can have several) in a batch.
 - Poisson distribution
 - c-diagram
 - u-diagram

Defect

- Non-conforming
 - Deviation from given specifications
- Defective
 - Defective unit
 - Can have one or more non-conformities.
- In this chapter: **Non-conforming**

Classification Demerit systems

page 301

- Class A – Very Serious
 - The unit is either completely unfit for service, or will fail in service in such a manner that cannot be easily corrected in the field, or will cause personal injury or property damage.
- Class B -- Serious
- Class C – Moderately Serious
- Class D Defects -- Minor

Fraction nonconforming

- Number of nonconforming divided with the total number of units in the population.
- A unit can have multiple characteristics. If at least one characteristic differs from specifications the unit is judged as nonconforming.
- Given as percentage.

Model for fraction nonconformities.

Assume that the process is *stable* and that the units are *independent* of each other.

Let

- n = sample size
- p = fraction nonconforming.
- D = number of defects in the batch.

D is then Binomially distributed and

$$P(D = x) = \binom{n}{x} p^x (1-p)^{n-x}, x = 0, 1, \dots, n$$

$$\begin{cases} \hat{p} = \frac{D}{n} \\ \mu_{\hat{p}} = p \\ \sigma_{\hat{p}}^2 = \frac{p(1-p)}{n} \end{cases}$$

Control limits for binomial data.

Phase I

m = number sample groups under phase I.

Control limits:

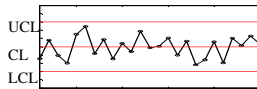
$$\hat{p}_i = \frac{D_i}{n}, i = 1, \dots, m$$

$$UCL = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

$$\bar{p} = \frac{\sum_{i=1}^m \hat{p}_i}{m} = \frac{\sum_{i=1}^m D_i}{m}$$

$$CL = \bar{p}$$

$$LCL = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$



Tid, provnummer

Control limits Binomial data

Probability limits

Exact Binomial

$$UCL \text{ Decide } UCL: P\left(\frac{D}{n} \leq UCL\right) \geq 1 - \frac{\alpha}{2}$$

$$LCL \text{ Decide } LCL: P\left(\frac{D}{n} \leq LCL\right) \leq \frac{\alpha}{2}$$

$$\text{Matlab: } UCL = 1/n \cdot \text{binoinv}(1 - \frac{\alpha}{2}, n, \bar{p})$$

Poisson approximation

$$\sum_{i=0}^D \binom{n}{x} p^x (1-p)^{n-x} \approx \sum_{i=0}^D e^{-np} \frac{(np)^x}{x!}$$

Normal approximation

For small values on p the probability limits should be used.

..... EXAMPLE 6-1

Frozen orange juice concentrate is packed in 6-oz cardboard cans. These cans are formed on a machine by spinning them from cardboard stock and attaching a metal bottom panel. By inspection of a can, we may determine whether, when filled, it could possibly leak either on the side seam or around the bottom joint. Such a nonconforming can has an improper seal on either the side seam or the bottom panel. We wish to set up a control chart to improve the fraction of nonconforming cans produced by this machine.

To establish the control chart, 30 samples of $n = 50$ cans each were selected at half-hour intervals over a three-shift period in which the machine was in continuous operation. The data are shown in Table 6-1.

Table 6-1 Data for Trial Control Limits, Example 6-1, Sample Size $n = 50$

Sample Number	Number of Nonconforming Cans, D_i	Sample Fraction Nonconforming, \hat{p}_i	Sample Number	Number of Nonconforming Cans, D_i	Sample Fraction Nonconforming, \hat{p}_i
1	12	0.24	17	10	0.20
2	15	0.30	18	5	0.10
3	8	0.16	19	13	0.26
4	10	0.20	20	11	0.22
5	4	0.08	21	20	0.40
6	7	0.14	22	18	0.36
7	16	0.32	23	24	0.48
8	9	0.18	24	15	0.30
9	14	0.28	25	9	0.18
10	10	0.20	26	12	0.24
11	5	0.10	27	7	0.14
12	6	0.12	28	13	0.26
13	17	0.34	29	9	0.18
14	12	0.24	30	6	0.12
15	22	0.44		347	$\bar{p} = 0.2313$
16	8	0.16			

Exempel 6-1 Apelsin juice boxes

Estimate process average

$$\left(\begin{matrix} m = 30 \\ n = 50 \end{matrix} \right)$$

$$\bar{p} = \frac{\sum_{i=1}^m D_i}{m \cdot n} = \frac{347}{30 \cdot 50} \approx 0.2313$$

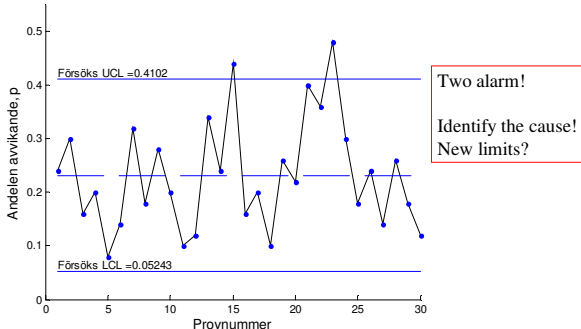
Control limits

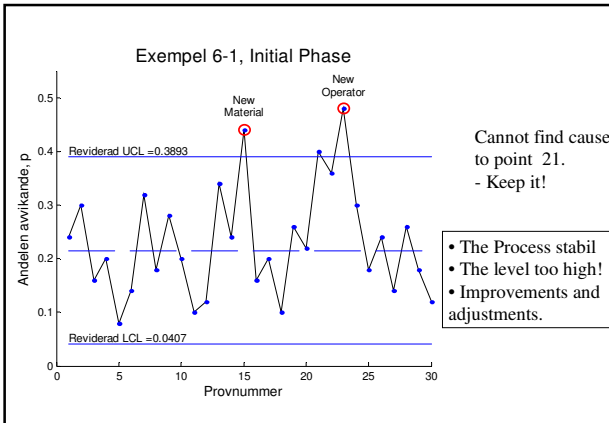
$$UCL = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.4102$$

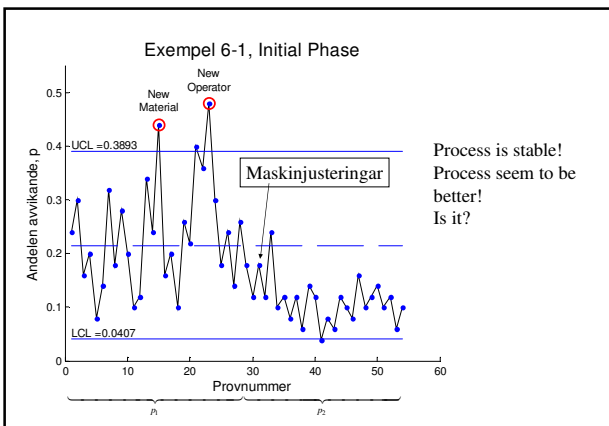
$$CL = \bar{p}$$

$$LCL = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.0524$$

Exempel 6-1, Initial Phase







Is the adjusted process better?

Test hypothesis

$$\begin{cases} H_0: p_1 = p_2 \\ H_1: p_1 > p_2 \end{cases}$$

Sample statistics (normal approx.):

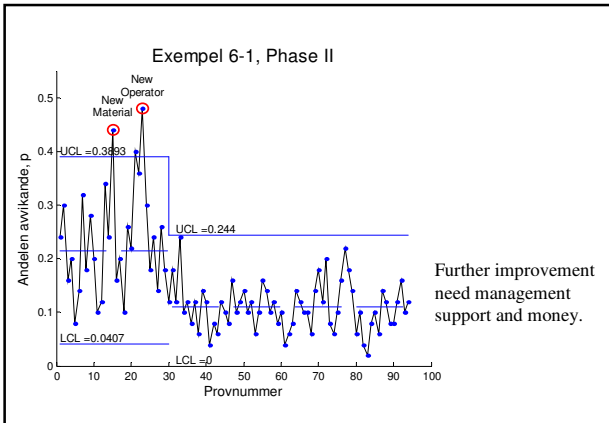
$$Z_0 = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} \sim N(0,1)$$

$$\hat{p} = \frac{n_1 \hat{p}_1 + n_2 \hat{p}_2}{n_1 + n_2}$$

$\hat{p}_1 = 0.2150$
 $\hat{p}_2 = 0.1108$
 $\hat{p} = 0.1669$

$$z_0 = \frac{0.2150 - 0.1108}{\sqrt{0.1669 \cdot 0.8331 \left(\frac{1}{1400} + \frac{1}{1200}\right)}} \approx 7.10 > z_{0.05} = 1.645$$

We reject H_0 on level $\alpha = 0.05$.
 Answer: Yes. There is a difference!



Exempel 6-1

Control limits (Normal approx)

$$UCL = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.2440$$

$$LCL = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = -0.0224 \rightarrow 0$$

Control limits (Binomial)

$$UCL = 0.26$$

$$LCL = 0$$

Wider limits!

Design of p-diagram

- sample size
- sample frequency
- width of control limits (α)

How large sample size (n)?

Choose n such that
 $P(D \geq 1 | n, p) \geq 0.95$

Exemple
 $p = 0.01$

$$P(D=0) = \binom{n}{0} p^0 (1-p)^{n-0} = (1-p)^n \leq 0.05$$

$$n = \frac{\log(0.05)}{\log(1-p)} \approx 298$$

We want for Ex. 6-1 that $LCL > 0$

$$LCL = p - L \sqrt{\frac{p(1-p)}{n}} > 0$$

$$n > L^2 \frac{1-p}{p} = 9 \frac{1-0.1108}{0.1108} \approx 72.2$$

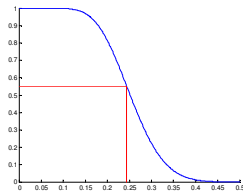
$n = 73$

OC-curve

$$\beta = P(\hat{p} < UCL | p) - P(\hat{p} < LCL | p) = P(D < nUCL | p) - P(D < nLCL | p)$$

$$ARL_0 = \frac{1}{\alpha}$$

$$ARL_1 = \frac{1}{1-\beta}$$



p-diagram

- Valid for all proportions nonconforming if
 - Failure frequency is constant.
 - Data are independent.
- Warning for
 - Cluster
 - Dependencies
 - ...

np-diagram

- Control the number of defect instead of proportion.
- Clearer!

$$UCL = np + 3\sqrt{np(1-p)}$$

$$CL = np$$

$$LCL = np - 3\sqrt{np(1-p)}$$

- Choose integers as limits as

Control chart for nonconformities

- A unit can have many nonconformities.
- Number of defects is important.
- Number of defects per unit.
 - Defects per car
- Number of defects per "area", "length"
 - Defects per m of pipeline
 - Number of pinholes per m² on plastic film

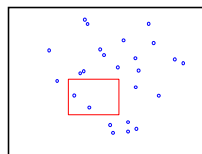
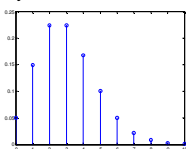
Poisson distribution

$D \sim \text{Poi}(c)$

$$p(x) = P(D=x) = \frac{e^{-c} c^x}{x!}, x=0,1,\dots$$

$c > 0$ the parameter of Poisson distribution

Not symmetrical!



Exemple holes in film

Poisson control chart

Estimate c with Control limits becomes

$$\hat{c} = \frac{\sum_{i=1}^m D_i}{m} \quad \begin{cases} UCL = \bar{c} + 3\sqrt{\bar{c}} \\ CL = \bar{c} \\ LCL = \bar{c} - 3\sqrt{\bar{c}} \end{cases}$$

Out of control plan (OCAP) must be written.

Classify defects.

Pareto diagram

Cause and effects diagram

Further Analysis of Nonconformities

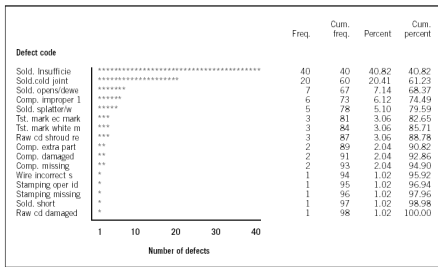


Figure 6-14 Pareto analysis of nonconformities for the printed circuit board process.

- Refer to Table 6-9 for occurrence of defect type by type of printed circuit board (part number)

Analyse the defects!

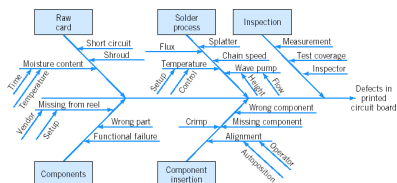


Figure 6-15 Cause-and-effect diagram.

u-diagram

- Sample size n
- Number of defects in sample D

$$u = \frac{D}{n}$$

$$\hat{u} = \frac{\sum_{i=1}^m D_i}{m \cdot n}$$

Control limits

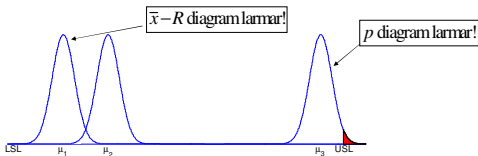
$$\begin{cases} UCL = \bar{u} + 3\sqrt{\frac{\bar{u}}{n}} \\ CL = \bar{u} \\ LCL = \bar{u} - 3\sqrt{\frac{\bar{u}}{n}} \end{cases}$$

Problem with Poisson model

- Cluster
 - Bacteria
 - Number of defects per car
- Compound Poisson distribution

To choose between variable and attribute diagram?

- Attribute: The sum of many defects collected in one diagram.
- Variable: More knowledge about the phenomenon.



6-5 GUIDELINES FOR IMPLEMENTING CONTROL CHARTS

Almost any process will benefit from SPC, including the use of control charts. In this section, we present some general guidelines helpful in implementing control charts. Specifically, we deal with the following:

1. Determining **which** process characteristics to control
2. Determining **where** the charts should be implemented in the process
3. Choosing the proper **type** of control charts
4. Taking **actions** to improve processes as the result of SPC/control chart analysis
5. Selecting data-collection systems and computer software

The guidelines are applicable to both variables and attributes control charts. Remember, control charts are not only for process surveillance; they should be used as an active, on-line method for reduction of process variability.

Determining Which Characteristics to Control and Where to Put the Control Charts

At the start of a control chart program, it is usually difficult to determine which product or process characteristics should be controlled and at which points in the process to apply control charts. Some useful guidelines follow.

1. At the beginning of a control chart program, control charts should be applied to any product characteristics or manufacturing operations believed to be important. The charts will provide immediate feedback as to whether they are actually needed.
2. The control charts found to be unnecessary should be removed, and others that engineering and operator judgment indicates may be required should be added. More control charts will usually be employed at the beginning than after the process has stabilized.
3. Information on the number and types of control charts on the process should be kept current. It is best to keep separate records on the variables and attributes charts. In general, after the control charts are first installed, we often find that the number of control charts tends to increase rather steadily. After that, it will usually decrease. When the process stabilizes, we typically find that it has the same number of charts from one year to the next. However, they are not necessarily the same charts.

4. If control charts are being used effectively and if new knowledge is being gained about the key process variables, we should find that the number of \bar{x} and R charts increases and the number of attributes control charts decreases.
5. At the beginning of a control chart program there will usually be more attributes control charts, applied to semifinished or finished units near the *end* of the manufacturing process. As we learn more about the process, these charts will be replaced with \bar{x} and R charts applied *earlier* in the process to the critical parameters and operations that result in nonconformities in the finished product. Generally, **the earlier that process control can be established, the better**. In a complex assembly process, this may imply that process controls need to be implemented at the vendor or supplier level.
6. Control charts are an on-line, process-monitoring procedure. They should be implemented and maintained as close to the work center as possible, so that feedback will be rapid. Furthermore, the process operators and process engineering should have direct responsibility for collecting the process data, maintaining the charts, and interpreting the results. The operators and engineers have the detailed knowledge of the process required to correct process upsets and use the control chart to improve process performance. Microcomputers can speed up the feedback and should be an integral part of any modern, on-line, process-control procedure.
7. The out-of-control-action plan (OCAP) is a vital part of the control chart. Operating and engineering personnel should strive to keep OCAPs up-to-date and valid.

Choosing the Proper Type of Control Chart

A. \bar{x} and R (or \bar{x} and s) charts. Consider using variables control charts in these situations:

1. A new process is coming on stream, or a new product is being manufactured by an existing process.
2. The process has been in operation for some time, but it is chronically in trouble or unable to hold the specified tolerances.
3. The process is in trouble, and the control chart can be useful for diagnostic purposes (troubleshooting).
4. Destructive testing (or other expensive testing procedures) is required.
5. It is desirable to reduce acceptance-sampling or other downstream testing to a minimum when the process can be operated in control.
6. Attributes control charts have been used, but the process is either out of control or in control but the yield is unacceptable.
7. There are very tight specifications, overlapping assembly tolerances, or other difficult manufacturing problems.
8. The operator must decide whether or not to adjust the process, or when a setup must be evaluated.
9. A change in product specifications is desired.
10. Process stability and capability must be continually demonstrated, such as in regulated industries.

B. **Attributes Charts (p charts, c charts, and u charts).** Consider using attributes control charts in these situations:

1. Operators control the assignable causes, and it is necessary to reduce process fall-out.
2. The process is a complex assembly operation and product quality is measured in terms of the occurrence of nonconformities, successful or unsuccessful product function, and so forth. (Examples include computers, office automation equipment, automobiles, and the major subsystems of these products.)
3. Process control is necessary, but measurement data cannot be obtained.
4. A historical summary of process performance is necessary. Attributes control charts, such as p charts, c charts, and u charts, are very effective for summarizing information about the process for management review.
5. Remember that attributes charts are generally inferior to charts for variables. Always use \bar{x} and R or \bar{x} and s charts whenever possible.

C. **Control Charts for Individuals.** Consider using the control chart for individuals in conjunction with a moving-range chart in these situations:

1. It is inconvenient or impossible to obtain more than one measurement per sample, or repeat measurements will only differ by laboratory or analysis error. Examples often occur in chemical processes.
2. Automated testing and inspection technology allow measurement of every unit produced. In these cases, also consider the cumulative sum control chart and the exponentially weighted moving average control chart discussed in Chapter 7.
3. The data become available very slowly, and waiting for a larger sample will be impractical or make the control procedure too slow to react to problems. This often happens in nonproduct situations; for example, accounting data may become available only monthly.
4. Generally, once we are in phase II, individuals charts have poor performance in shift detection and can be very sensitive to departures from normality. Always use the EWMA and cusum charts of Chapter 8 in phase II instead of individuals charts whenever possible.

Actions Taken to Improve the Process

Figure 6-27 gives the answers to two questions: "Is the process in control?" and "Is the process capable (in the sense of the previous paragraph)?" Each of the four cells in the figure contains some recommended courses of action that depend on the answers to these two questions.

		IS THE PROCESS CAPABLE?	
		Yes	No
IS THE PROCESS IN CONTROL?	Yes	SPC	SPC Experimental design Investigate specifications Change process
	No	SPC	SPC Experimental design Investigate specifications Change process

Figure 6-27 Actions taken to improve a process.

The lower two boxes in Fig. 6-27 deal with the case of an out-of-control process. The southeast corner presents the case of a process that is out of control and not capable. (Remember our nontechnical use of the term *capability*.) The actions recommended here are identical to those for the box in the northeast corner, except that SPC would be expected to yield fairly rapid results now, because the control charts should be identifying the presence of assignable causes. The other methods of attack will warrant consideration and use in many cases, however. Finally, the southwest corner treats the case of a process that exhibits lack of statistical control but does not produce a meaningful number of defectives because the specifications are very wide. SPC methods should still be used to establish control and reduce variability in this case, for the following reasons:

1. Specifications can change without notice.
2. The customer may require both **control** and **capability**.
3. The fact that the process experiences assignable causes implies that unknown forces are at work; these unknown forces could result in poor capability in the near future.
