Lektion 6

2007-12-06_2
Chapter 7
Process capability

Process capability Chapter 7

Some tools:

- Histogram
- Probability plots
- Control charts
- Designed experiments
- ….

Capability indices

- C_p
- C_{pk}
- C_{pu}
- C_{pl}
- P_p
- ….

Process capability

- Statistical process control – stability
- Tolerance limits – Customer requirements
- Capability (duglighet)
  - Connects SPC with requirements.

- *The process ability to produce units fulfilling the customer requirements.*

Capability

- Is the process capable to fulfil customer requirements?
- Tolerance limits?
- Stability?
- Independence?
- Distribution?

Stability is the most important factor!

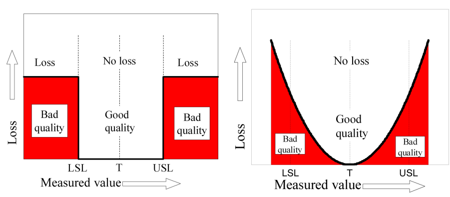
Process capability

- Systematic variation – SPC
- Random variation - capability

		Process is stable?	
		No	Yes
Fulfil spec.?	No	No!	?
	Yes	?	Yes!

Comment:
Stability is a prerequisite.
In square No – Yes (Not stab. Spec.) the instability must be "in control".

Tolerance limits



Question tolerance limits!
Ask what the limits means.
To narrow limits makes the product more expensive.

Capability

- What is the process capable of?

The proces "natural" tolerance limits:

$UNTL = \mu + 3\sigma$

$LNTL = \mu - 3\sigma$

0.27% sounds little, but
- locked out 4 times per year.
- bad drinking-water 6 times/year.
- no electricity ….

Obs: 0.27% is under normal distribution

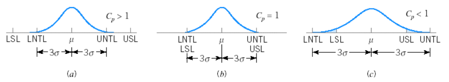
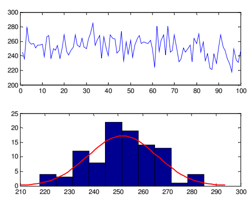


Figure 5-3 Process fallout and the process capability ratio C_p.

Process capability study

1. Prediction of how well the process will fulfil the tolerance limits.
2. Aids the designers to choose/modify the production process.
3. Aids when deciding sampling frequency at SPC.
4. Requirements on new equipment
5. Choosing between suppliers
6. Planning of production where process and tolerances covariates.
7. Decreasing the variation in manufacturing process.

Tool histogram



Rather normally distributed
99.73% between
210 and 293.5

245	239	246	263	269	238	224	250	270	269
236	266	241	270	264	255	281	232	245	252
279	268	247	285	263	264	246	235	259	247
260	237	252	254	244	268	261	251	233	236
256	250	269	261	248	257	269	245	243	229
257	248	255	268	274	259	241	249	223	218
251	254	252	237	233	258	281	255	221	249
255	237	264	266	259	257	245	232	242	234
255	251	252	249	242	261	247	264	230	231
256	269	250	240	261	249	269	257	252	247

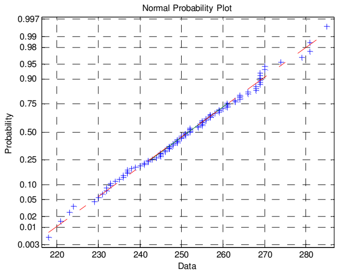
$\bar{x} = 251.77$

$s = 13.91$

Process capability $(\bar{x} \pm 3s)$:

251.8 ± 41.7

Tool: Probability plot



Estimate σ with:

$\hat{\sigma} = 84\,perc. - 50\,perc. =$
$= 267 - 252 = 15$

Very sensitive for distribution assumption!
Test distribution with some test.

Capability indice C_p

$$C_p = \frac{USL - LSL}{6\sigma}$$

Is estimated by the within group variation

$$\hat{\sigma} = \frac{\bar{s}}{c_4} \text{ or } \hat{\sigma} = \frac{\bar{R}}{d_2}$$

$$\hat{C}_p = \frac{USL - LSL}{6\hat{\sigma}}$$

Three **very** important conditions:
The process **must**:
1. be in statistical control (stable)
2. have a normally distributed outcome
3. have independent outome.

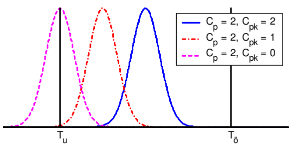
Misuse of index is very common in industry.
A process is often more complex and cannot be described with only one number.
Erronous decisions cost lots of money!
(Investments, choice of supplier…)

Actual capability index: C_{pk}

- C_p does not take into account where process mean is.
- C_p does not give the proportion outside tolerance limits.
- A new index was invented:

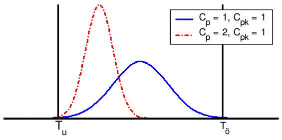
$$C_{pk} = \min\left(\frac{USL - \mu}{3\sigma}, \frac{\mu - LSL}{3\sigma}\right)$$

C_p and C_{pk}



Fraction outside tolerance limits:

$$p = \Phi\left(-3C_{pk}\right) + \Phi\left(-3\left(2C_p - C_{pk}\right)\right)$$



Alternative index: ppm (parts per million)

- Calculate fraction outside tolerance limits:

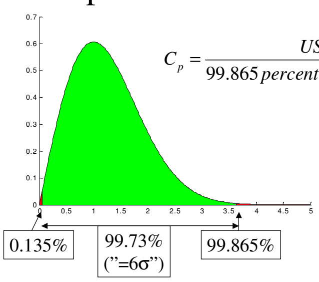
$$p = P(\text{outside tolerance limits})$$

$$p = \Phi\left(-3C_{pk}\right) + \Phi\left(-3\left(2C_p - C_{pk}\right)\right) \quad \text{(Normal data)}$$

		C_p						
		0	0.33	0.67	1.00	1.33	1.67	2.00
	0	1.00	0.523	0.5	0.5	0.5	0.5	0.5
	0.33	–	0.317	0.160	0.159	0.159	0.159	0.159
	0.67	–	–	4.55×10^{-2}	2.28×10^{-2}	2.28×10^{-2}	2.28×10^{-2}	2.28×10^{-2}
C_{pk}	1.00	–	–	–	2.70×10^{-3}	1.35×10^{-3}	1.35×10^{-3}	1.35×10^{-3}
	1.33	–	–	–	–	6.33×10^{-5}	3.17×10^{-5}	3.17×10^{-5}
	1.67	–	–	–	–	–	5.73×10^{-7}	2.87×10^{-7}
	2.00	–	–	–	–	–	–	1.97×10^{-9}

Alternative index: percentile method

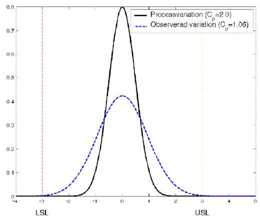
$$C_p = \frac{USL - LSL}{99.865\,percentile - 0.135\,percentile}$$

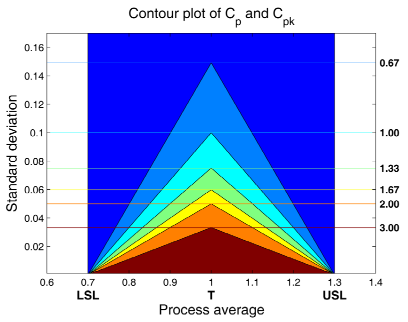


Measurements

- SPC and Capability need good measurement quality!

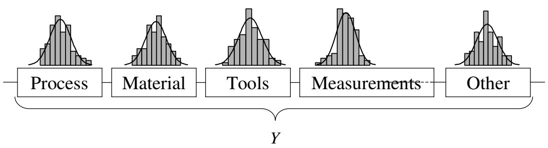
Wrong conclusion about the process due to bad measurement system.





Better interpretation of capability?

Tolerance chain



$$Y = X_{\text{Process}} + X_{\text{Material}} + X_{\text{Tools}} + X_{\text{Measurements}} + \ldots + X_{\text{Other}}$$

$$Var[Y] = \sigma^2_{\text{Process}} + \sigma^2_{\text{Material}} + \sigma^2_{\text{Tool}} + \sigma^2_{\text{Measurements}} + \ldots + \sigma^2_{\text{Others}}$$

Improve capability

- Design of experiments
- Robust design
- Widen the tolerance limits
- Decrease the capability requirement
- Process control.

Capability requirement

- ”Ordinary” capability: $\begin{cases} C_p = \dfrac{T_{Ö} - T_U}{6\sigma} \\ C_{pk} = \min\left(\dfrac{T_{Ö} - \mu}{3\sigma}, \dfrac{\mu - T_U}{3\sigma}\right) \end{cases}$
- Customer requirement: $C_p \geq 1.33$
- How should the company show this?

What sample size?

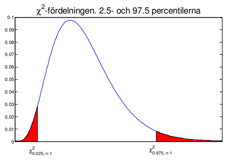
- 320 measurements from a stable process.
- Assume independent and normally distributed data.
- Calculate 90% confidence interval:

$$\hat{C}_p \sqrt{\frac{\chi^2_{(1-\frac{\alpha}{2}, n-1)}}{n-1}} \leq C_p \leq \hat{C}_p \sqrt{\frac{\chi^2_{(\frac{\alpha}{2}, n-1)}}{n-1}}$$

$$\hat{C}_{pk}\left(1 - z_{\frac{\alpha}{2}}\sqrt{\frac{1}{9n\hat{C}_{pk}} + \frac{1}{2(n-1)}}\right) \leq C_{pk} \leq \hat{C}_{pk}\left(1 + z_{\frac{\alpha}{2}}\sqrt{\frac{1}{9n\hat{C}_{pk}} + \frac{1}{2(n-1)}}\right)$$

Confidence interval for C_p

$$C_p = \frac{USL - LSL}{6\sigma}$$



Proof:

$$\frac{(n-1)s^2}{\sigma^2} \sim \chi^2_{n-1}$$

dvs

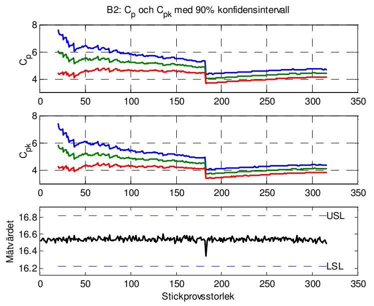
$$\chi^2_{\alpha/2,n-1} \le \frac{(n-1)s^2}{\sigma^2} \le \chi^2_{1-\alpha/2,n-1}$$

$$\frac{1}{s}\sqrt{\frac{\chi^2_{\alpha/2,n-1}}{n-1}} \le \frac{1}{\sigma} \le \frac{1}{s}\sqrt{\frac{\chi^2_{1-\alpha/2,n-1}}{n-1}}$$

$$\frac{USL-LSL}{6s}\sqrt{\frac{\chi^2_{\alpha/2,n-1}}{n-1}} \le \frac{USL-LSL}{6\sigma} \le \frac{USL-LSL}{6s}\sqrt{\frac{\chi^2_{1-\alpha/2,n-1}}{n-1}}$$

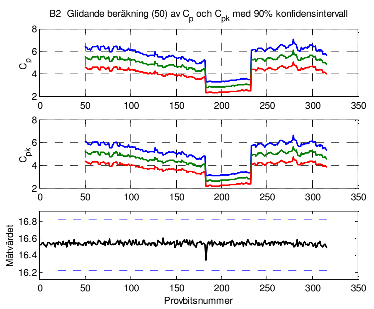
$$\hat{C}_p\sqrt{\frac{\chi^2_{\alpha/2,n-1}}{n-1}} \le C_p \le \hat{C}_p\sqrt{\frac{\chi^2_{1-\alpha/2,n-1}}{n-1}}$$

How large sample size?



Observe:
One value disturbes!
Stable process is necessary

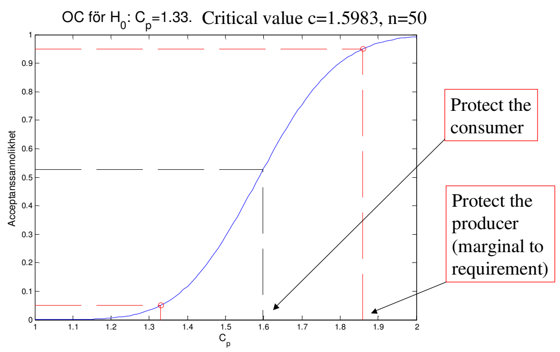
Stability of capability



Here: Big difference where to take the sample!

Test of capability
n=50, α=0.05, β=0.05.

- Requirement: $C_p>1.33$
- Sample size n=50
- Accept if $\hat{C}_p \geq c$
- Consumer risk => c=1.60
- Producer risk => process requirement: $C_{pk} \geq 1.86$



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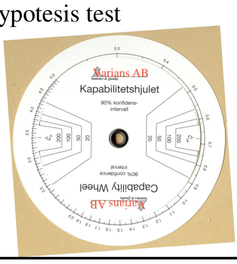
Provstorlek	$\alpha=\beta=0.10$: $C_p(High)/C_p(Low)$	$\alpha=\beta=0.10$: $C_p/C_p(Low)$	$\alpha=\beta=0.05$: $C_p(High)/C_p(Low)$	$\alpha=\beta=0.05$: $C_p/C_p(Low)$
10	1.88	1.47	2.26	1.65
20	1.53	1.28	1.73	1.37
30	1.41	1.21	1.55	1.28
40	1.34	1.18	1.46	1.23
50	1.30	1.15	1.40	1.20
60	1.27	1.14	1.36	1.18
70	1.25	1.13	1.33	1.16
80	1.23	1.12	1.30	1.15
90	1.21	1.11	1.28	1.14
100	1.20	1.10	1.26	1.13

Exemple:
$n=50$
$\alpha=0.05$
$\beta=0.05$
$C_{p0}=1.33$

Acceptance number
$c=1.20\cdot C_{p0}=1.596$

The capability wheel (©Varians)

- Confidence intervals for C_p and C_{pk}.
- Hypotesis test



Obs!

- Normally distributed data!
- Independent outcomes!
- Stable process!

Capability - difficulties

- Non-normal distributions
- Ignorant customers and suppliers.
- What capability indice should be used?
  - P_p, P_{pk}, C_p, C_{pk}?
- How should the standard deviation be estimated?
  - Range or s-method?
- To hard data reduction?
  - Do not forget histogram and control charts!
  - Is the index a useful model of the process?

P_p and P_{pk}

$$\hat{P}_p = \frac{USL - LSL}{6s}$$

$$s = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}}$$

$P_p = C_p$ if the process is stable

P_p and P_{pk} cannot be interpreted for a non-stable process.

s is estimated from all variation, even the systematic.

Used in AIAG, ANSI.

Statistical terrorism!

Does not tell us anything of the future!

Exempel på en plan för att säkerställa processens kapabilitet.