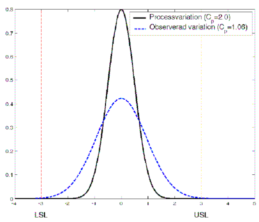
Lektion 6

2007-12-06_3
Chapter 7
Measurement systems analysis

Measurement system!

- Statistical process control requires measurement of good quality!



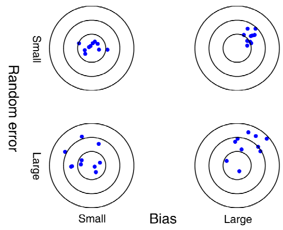
Wrong conclusion about the process due to measurement error!
- Very expensive!

Measurement systems analysis

- Make well-founded decisions
  - Model quality
  - Data
- Measurement error compared to product variation?
- Measurement error compared to tolerance limits?
- Measurement systems analysis can be complex.
- Measurement system covariates with the product.

Different types of measurement errors.

- Bias (Missvisning)
- Spread



- Big errors
  - Wrong product
  - Write wrong
  - …

A simple model of the measurement system

Model

$y = x + \varepsilon$

y is the observed value.

x is the true value.

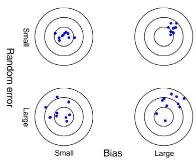
ε is the measurement error.

Bias:

$E[\varepsilon] \neq 0$

Spread:

$V[\varepsilon] > \sigma_0^2$



Variation model

$\sigma_{Total}^2 = \sigma_P^2 + \sigma_{Gauge}^2$

Requirement:

$\frac{P}{T} = \frac{k\hat{\sigma}_{Gauge}}{USL - LSL} \leq 0.10,$ (Process capability)

$\hat{\rho}_M = \frac{\sigma_{Gauge}^2}{\sigma_{Total}^2}$

Some requirements on a measurement system.

1. The system must have adequate discrimination and sensitivity. The increments should be small relative the process variatoin or specification limits. Thumb rule of 10: states that instrument discrimination shoudl divide the tolerance (or process) into ten parts or more.
2. The measurement system ought to be in statistical control. That means that under repeatable conditions, the variation in the measurement system is due to common causes only.
3. For *product control,* variability of the measurement system must be small compared to the specification limits.
4. For *process control,* the variability of the measurement system ought to demonstrate effective resolution and be small compared to manufacturing process variation.

Requirements on measurement system

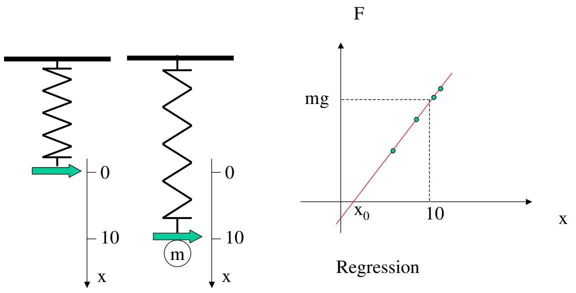
Example of requirement:

$$\frac{P}{T} = \frac{k\hat{\sigma}_{Gauge}}{USL - LSL} \quad \text{(Process capability)}$$

$$\hat{\rho}_M = \frac{\sigma^2_{Gauge}}{\sigma^2_{Total}} \quad \text{(Process variation)}$$

Problem:
If the process variation decreases then
the measurement system may be rejected.

Calibration of measurement system



Control charts on the measurement system instead of calibration?

- Calibration – control on individual measurements.
- SPC to detect changes.
- Exemple: Measure a reference weight each day and plot it in a control chart.
  - Shewhart, Cusum or EWMA?

MSA
Some important characteristics

- Stability
  - Control chart
- Bias
  - Measure a reference many times.
- Linjaritet
- Repeatability and reproducability
  - ”Gauge R&R”
  - ANOVA

Repeteatability and reproducability
(M. Arnér: *Mätosäkerhet*)

- **Repeteatability**: Repeatability or lack of repeatability is the variation that comes when the same operator uses the same measurement equipment and measures the same unit many times
- **Reproducability**: Reproducability or lack of reproducability is used for the variation that comes when e.g.
  - different operators with the same measurement equipment measures hte same unit *or*
  - one operator is measuring the same unint with different measurement equipment.

$$\sigma^2_{\text{Measurement error}} = \sigma^2_{Gauge} = \sigma^2_{\text{Repeatability}} + \sigma^2_{\text{Reproducibility}}$$

Gauge R&R

$y_{ijk} = \mu + P_i + O_j + (PO)_{ij} + \varepsilon_{ijk}$

$P_i, O_j, (PO)_{ij}, \varepsilon_{ijk}$ are independent and normally distributed with average 0

$V(P_i) = \sigma_P^2$

$V(O_j) = \sigma_O^2$

$V((PO)_{ij}) = \sigma_{PO}^2$

$V(\varepsilon_{ijk}) = \sigma^2$

$V(y_{ijk}) = \sigma_P^2 + \sigma_O^2 + \sigma_{PO}^2 + \sigma^2$

$\sigma_P^2, \sigma_O^2, \sigma_{PO}^2, \sigma^2$ are called variance components.

ANOVA is used in the analysis.

Exemple (table 7.7)

	Inspector 1			Inspector 2			Inspector 3		
Detalj nr:	Test 1	Test 2	Test 3	Test 1	Test 2	Test 3	Test 1	Test 2	Test 3
1	37	38	37	41	41	40	41	42	41
2	42	41	43	42	42	42	43	42	43
3	30	31	31	31	31	31	29	30	28
4	42	43	42	43	43	43	42	42	42
5	28	30	29	29	30	29	31	29	29
6	42	42	43	45	45	45	44	46	45
7	25	26	27	28	28	30	29	27	27
8	40	40	40	43	42	42	43	43	41
9	25	25	25	27	29	28	26	26	26
10	35	34	34	35	35	34	35	34	35

The sums of squares (see chap. 12-4.2)

$$SS_{Total} = SS_{Parts} + SS_{Operators} + SS_{PO} + SS_{Error}$$

Gauge R&R

Divide the sums of squares with the degrees of freedom:

$$MS_P = \frac{SS_P}{p-1} = \frac{3935.96}{9} = 437.328$$

$$MS_O = \frac{SS_O}{o-1} = \frac{39.27}{2} = 19.633$$

$$MS_{PO} = \frac{SS_{PO}}{(p-1)(o-1)} = \frac{48.51}{9 \cdot 2} = 2.695$$

$$MS_E = \frac{SS_E}{po(n-1)} = \frac{30.67}{10 \cdot 3 \cdot 2} = 0.511$$

ANOVA in Matlab

ANOVA Table

Source	SS	df	MS	F	Prob>F
Columns	3935.96	9	437.328	855.64	0
Rows	39.27	2	19.633	38.41	0
Interaction	48.51	18	2.695	5.27	0
Error	30.67	60	0.511		
Total	4054.4	89			

Command: anova2

Estimate the variance components

$\hat{\sigma}^2 = MS_E = 0.51$

$\hat{\sigma}^2_{PO} = \frac{MS_{PO} - MS_E}{n} = 0.73$

$\hat{\sigma}^2_O = \frac{MS_O - MS_{PO}}{pn} = 0.56$

$\hat{\sigma}^2_P = \frac{MS_P - MS_{PO}}{on} = 48.29$

If $\hat{\sigma}^2_{PO} < 0$ then choose a model without interactions:

$y_{ijk} = \mu + P_i + O_j + \varepsilon_{ijk}$

Repeteatability and reproducability

$\sigma^2_{\text{Repeatability}} = \sigma^2 = 0.51$

$\sigma^2_{\text{Reproducibility}} = \sigma^2_O + \sigma^2_{PO} = 1.29$

$\sigma^2_{\text{Gauge}} = \sigma^2_{\text{Reproducibility}} + \sigma^2_{\text{Repeatability}} = 1.80$

$(LSL = 18,\ USL = 58)$

$\widehat{P/T} = \frac{6\hat{\sigma}_{Gauge}}{USL - LSL} = \frac{6 \cdot 1.34}{58 - 18} = 0.27$ Är större än 0.10!

Conclusion: We should improve the measurement system to be able to measure the product variation.