Solutions to Statistical Quality Control (MVE145/MSG600) 071218

Problem 1: The magnificent seven page 148 o 169

1. Histogram
2. Check sheet
3. Pareto chart
4. Cause-and effect diagram
5. Defect concentration diagram
6. Scatter diagram
7. Control chart

Problem 2:

1) **The main purpose of a control chart** is to improve the process. Most processes do not operate in a state of statistical control. Consequently, the routine and attentive use of control charts will identify assignable causes. If these causes can be eliminated from the process, variability will be reduced and the process will be improved.

2**) In phase I**, a set of process data is gathered and analyzed all at once in a retrospective analysis, constructing trial control limits to determine if the process has been in control over the period of time where the data were collected, and to see if reliable control limits can be established to monitor future production. The control chart in phase I is used to assist operating personnel in bringing the process into a state of statistical control.
In phase II, we use the control chart to monitor the process by comparing the sample statistic for each successive sample to the control limits.

3) **Systematic variation** is called assignable causes and usually arises from three sources: improperly adjusted or controlled machines, operator errors, or defective raw material.
Random variation is also called chance causes or background noise. It could be the temperature in the room, humidity, wear of a tool and so on.

4) **G R&R** means Gauge Repeatability and Reproducibility study. It is a designed factorial experiment because each operator measures all the parts. The analysis of variance is used to analyze the experiment and to estimate the appropriate components of measurement system variability for instance due to the operator, the parts and the interaction between these two.

5) **OCAP** means Out-Of-Control-Action Plan. An OCAP is a flow chart or text-based description of the sequence of activities that must take place following the occurrence of an activating event such as out-of-control signals. Developing an effective system for corrective action is an essential component of an effective SPC implementation.

6) $P(\text{false alarm}) = 1 - P(-3 < Z < 3) = 1 - [P(Z < 3) - (1 - P(Z < 3)] =$

$$= 2[1 - P(Z < 3)] = 0.0027$$

7) **ARL** (average run length) means the average number of points that must be plotted before a point indicates an out-of-control condition. In question 6

$$\text{ARL} = \frac{1}{P(\text{false alarm})} = \frac{1}{0.0027} = 370$$

8) **Conditions for using capability indices C_p and C_{pk}** are

a) The quality characteristic has a normal distribution
b) The process is in statistical control

9) $y_{ijk} = \mu + s_i + t_j + s_i \times t_j + \varepsilon_{ijk}$

i = the “i”th student
j = the “j”th teacher
k = the “k”th question asked

y = you result on a problem
μ = average knowledge (or average result)
s = knowledge due to student
t = how a teacher judge your answer
s x t = interaction term
ε = which question that is asked

Problem 3: N = 7600 AQL = 0.40% MILSTD105E Insp Level II

a)

Code letters ISO 2859

Lot or Batch Size	Special Inspection Levels				General Inspection Levels		
	S-1	S-2	S-3	S-4	I	II	III
2-8	A	A	A	A	A	A	B
9-15	A	A	A	A	A	B	C
16-25	A	A	B	B	B	C	D
26-50	A	B	B	C	C	D	E
51-90	B	B	C	C	C	E	F
91-150	B	B	C	D	D	F	G
151-280	B	C	D	E	E	G	H
281-500	B	C	D	E	F	H	J
501-1200	C	C	E	F	G	J	K
1201-3200	C	D	E	G	H	K	L
3201-10000	C	D	F	G	J	L	M
10001-35000	C	D	F	H	K	M	N
35001-150000	D	E	G	J	L	N	P
150001-500000	D	E	G	J	M	P	Q
500001-	D	E	H	K	N	Q	R

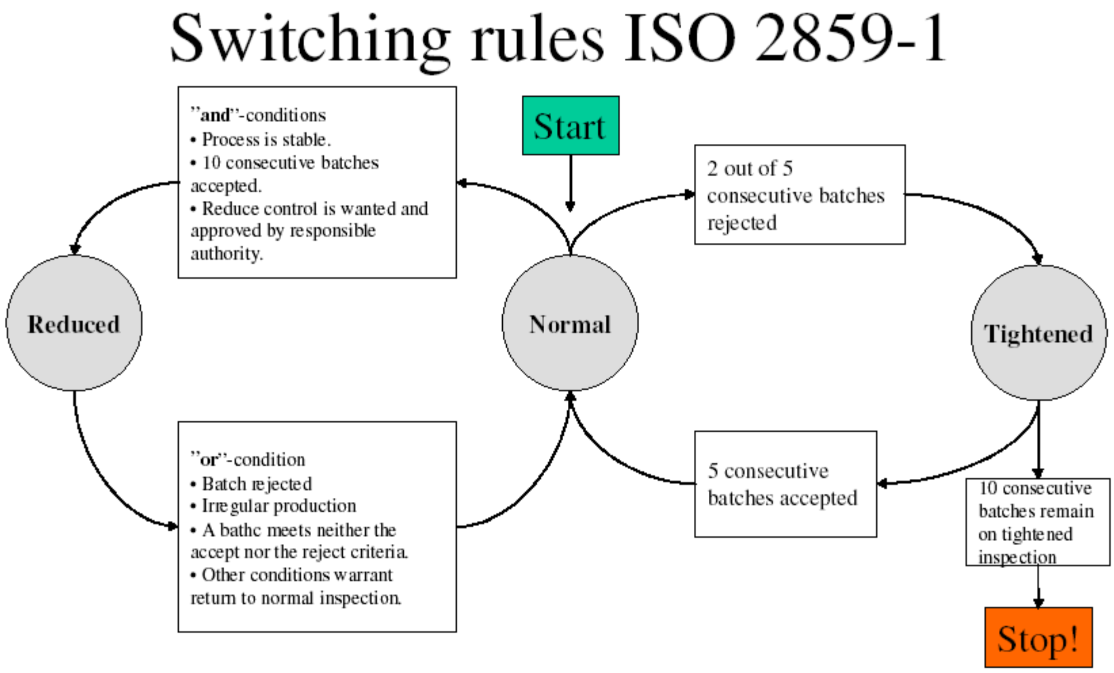
N = 7600 $\Rightarrow$ Letter L

1) Normal inspection p = 0.40 $\Rightarrow$ n = 200 c = 2 r = 3
2) Tightened inspection n = 200 c = 1 r = 2
3) Reduced inspection n = 80 c = 1 r = 3
4) The switching rules are

Start with normal inspection
- If 2 out of 5 consecutive lots are rejected go to tightened
- If 10 consecutive lots are accepted go to reduced

Reduced inspection
- Lot rejected go to normal inspection
- Irregular production go to normal inspection
- …
- …

Tightened inspection
- 5 consecutive lots accepted go to normal inspection
- 10 consecutive lots remain on tightened inspection discontinue inspection



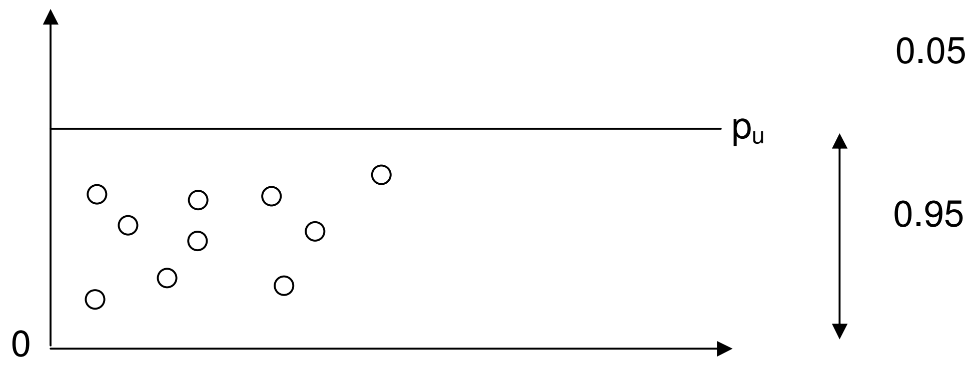
b) **100 % inspection** is often used in situations where the component is extremely critical and passing any defectives would result in an unacceptably high failure cost at subsequent stages, or where the supplier's process capability is inadequate to meet specifications.

Acceptance sampling is most likely to be useful in the situations described on page 647.

c) If you would have got some defectives then you could estimate the fraction defectives by using the formula $\hat{p} = d/n$. Then the upper 95% confidence limit had been calculated as

$$p_u = \hat{p} + 1.96\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

But now the number of defectives are 0.



A 95% confidence limit and normal inspection (n = 200) $\Rightarrow$

$P(\xi = 0) = (1 - p_u)^n = 0.05 \quad \Rightarrow \quad (1 - p_u)^{200} = 0.05 \quad \Rightarrow$

$\Rightarrow \quad p_u = 1 - 0.05^{1/200} \approx 0.015 \quad (1.5\ \%)$

Actually this is a binomial distribution $P(\xi = 0) = \binom{n}{0} p^0 (1 - p_u)^n$

Problem 4:

The sample size, n = 5 The number of samples, m = 25

Samples are taken every half hour.

$$\sum_{i=1}^{m} \bar{x}_i = 662.5 \quad \Rightarrow \quad \bar{\bar{x}} = \frac{662.5}{25} = 26.5$$

$$\sum_{i=1}^{m} R_i = 9.00 \quad \Rightarrow \quad \bar{R} = \frac{9.00}{25} = 0.36$$

a) Normal tolerance limits $\mu \pm 3\,\sigma$

estimated by $\bar{\bar{x}} \pm 3 \cdot \frac{\bar{R}}{d_2}$

Since n = 5 $\Rightarrow$ $d_2 = 2.326$ $\Rightarrow$ $\hat{\sigma} = \frac{0.36}{2.326} \approx 0.1548$

$26.5 \pm 3 \cdot 0.1548$

26.5 ± 0.46

b) Control limits

For the x-chart $\bar{\bar{x}} \pm A_2 \bar{R}$ Since n = 5 $\Rightarrow$ $A_2 = 0.577$

UCL = $\bar{\bar{x}} + A_2 \bar{R} = 26.5 + 0.577 \cdot 0.36 \approx 26.71$

CL = $\bar{\bar{x}} = 26.5$

LCL = $\bar{\bar{x}} - A_2 \bar{R} = 26.5 - 0.577 \cdot 0.36 \approx 26.29$

For the R-chart Since n = 5 $\Rightarrow$ $D_3 = 0$ and $D_4 = 2.114$

UCL = $D_4 \bar{R} = 2.114 \cdot 0.36 \approx 0.761$

CL = $\bar{R} = 0.36$

LCL = $D_3 \bar{R} = 0$

c) Fraction non-conforming i.e. outside the control limits 26.40 ± 0.5

Use the estimated μ- and σ-values

$p = P(\xi > 26.9) + P(\xi < 25.9) =$

$$= 1 - P(Z < \frac{26.9 - 26.5}{0.1548}) + P(Z < \frac{25.9 - 26.5}{0.1548}) =$$

$$= 1 - P(Z < 2.58) + P(Z < -3.86) = 1 - 0.9951 - (1 - 0.9999) \approx 0.0051$$

d) $ATS = ARL \cdot h \qquad h = 0.5$

$$p = P(LCL < \xi < UCL) = P(Z < \frac{26.71 - 26.4}{0.1548}) - P(Z < \frac{26.26 - 26.4}{0.1548}) =$$

$$= P(Z < 4.36) - P(Z < -1.60) = 1.000 - (1 - 0.9452) = 0.9452$$

$$ARL = \frac{1}{1-p} = \frac{1}{1-0.9452} \approx 18.248 \quad \Rightarrow \quad ATS = 18.248 \cdot 0.5 = 9.12 \text{ h}$$

Problem 6: ξ_i are $N(\mu = 98, \sigma = 8)$ $\quad n = 5$

UCL = 104
CL = 100
LCL = 96

$$P(\text{no alarm}) = P(LCL < \xi < UCL) = P(Z < \frac{104 - 98}{8/\sqrt{5}}) -$$

$$- P(Z < \frac{96 - 98}{8/\sqrt{5}}) \approx P(Z < 1.68) - P(Z < -0.56) \approx$$

$$\approx 0.9535 - (1 - 0.712) = 0.6655$$

$P(\text{alarm after max 3 samples}) = 1 - 0.6655^3 = 0.7053$

Problem 7: $\sum_{i=2}^{20} MR_i = 442 \quad \Rightarrow \quad \overline{MR} = \frac{442}{19} = 23.2632$

a)

$\hat{\sigma} = \frac{\overline{R}}{d_2}$ Since $n = 2 \Rightarrow d_2 = 1.128 \Rightarrow \hat{\sigma} = \frac{23.2632}{1.128} \approx 20.62$

b) $\mu_0 = 500$ $\lambda = 0.1$ $L = 2.7$ $z_i = \lambda x_i + (1 - \lambda) z_{i-1}$

$i = 1$ $z_1 = 0.1 \cdot 473 + 0.9 \cdot 500 = 497.3$

$i = 2$ $z_2 = 0.1 \cdot 512 + 0.9 \cdot 497.3 = 498.77$

$i = 3$ $z_3 = 0.1 \cdot 518 + 0.9 \cdot 498.77 = 500.693$

$i = 4$ $z_4 = 0.1 \cdot 492 + 0.9 \cdot 501 = 500$

$i = 5$ $z_5 = 0.1 \cdot 484 + 0.9 \cdot 500 = 498$

$i = 6$ $z_6 = 0.1 \cdot 512 + 0.9 \cdot 498 = 500$

$i = 7$ $z_7 = 0.1 \cdot 513 + 0.9 \cdot 500 = 501$

$i = 8$ $z_8 = 0.1 \cdot 536 + 0.9 \cdot 501 = 504$

$i = 9$ $z_9 = 0.1 \cdot 481 + 0.9 \cdot 504 = 502$

$i = 10$ $z_{10} = 0.1 \cdot 533 + 0.9 \cdot 502 = 505$

$i = 11$ $z_{11} = 0.1 \cdot 536 + 0.9 \cdot 505 = 508$

$i = 12$ $z_{12} = 0.1 \cdot 538 + 0.9 \cdot 508 = 511$

$i = 13$ $z_{13} = 0.1 \cdot 539 + 0.9 \cdot 511 = 514$

$i = 14$ $z_{14} = 0.1 \cdot 523 + 0.9 \cdot 514 = 515$

$i = 15$ $z_{15} = 0.1 \cdot 535 + 0.9 \cdot 515 = 517$

$i = 16$ $z_{16} = 0.1 \cdot 513 + 0.9 \cdot 517 = 517$

$i = 17$ $z_{17} = 0.1 \cdot 553 + 0.9 \cdot 517 = 520$

$i = 18$ $z_{18} = 0.1 \cdot 544 + 0.9 \cdot 520 = 523$

$i = 19$ $z_{19} = 0.1 \cdot 585 + 0.9 \cdot 523 = 529$

$i = 20$ $z_{20} = 0.1 \cdot 527 + 0.9 \cdot 529 = 529$

Control limits in the EWMA diagram:

$$UCL = \mu_0 + L\sigma\sqrt{\frac{\lambda}{2-\lambda}[1-(1-\lambda)^{2i}]} = 500 + 55.674\sqrt{\frac{1}{19}[1-(0.9)^{2i}]}$$

$$CL = \mu_0 = 500$$

$$LCL = \mu_0 - L\sigma\sqrt{\frac{\lambda}{2-\lambda}[1-(1-\lambda)^{2i}]} = 500 - 55.674\sqrt{\frac{1}{19}[1-(0.9)^{2i}]}$$

$i = 1$	$\Rightarrow$	UCL = 500 + 5.57 ≈ 506	LCL = 494
$i = 2$	$\Rightarrow$	UCL = 500 + 7.4902 ≈ 507	LCL = 493
$i = 3$	$\Rightarrow$	UCL = 500 + 8.743 ≈ 509	LCL = 491
$i = 4\text{-}5$	$\Rightarrow$	UCL = 510	LCL = 490
$i = 6\text{-}7$	$\Rightarrow$	UCL = 511	LCL = 489
$i = 8\text{-}14$	$\Rightarrow$	UCL = 512	LCL = 488
$i = 15\text{-}20$	$\Rightarrow$	UCL = 513	LCL = 487

Alarm in sample 13

Problem 8:

$$C_p = \frac{USL - LCL}{6\sigma} \qquad \text{Target value} = \frac{USL + LCL}{2}$$

$$\mu < T \quad \Rightarrow \quad C_{pk} = \frac{\mu - LSL}{3\sigma}$$

$$\mu \geq T \quad \Rightarrow \quad C_{pk} = \frac{USL - \mu}{3\sigma}$$

$$p = 1 - P(LSL < \xi < USL) = 1 - P(Z < \frac{USL-\mu}{\sigma}) + P(Z < \frac{LSL-\mu}{\sigma})$$

Choose one of them ($\mu \geq T$). Symmetry gives the same result for the other alternative.

1) $P(Z < \frac{USL - \mu}{\sigma}) = P(Z < \frac{3(USL - \mu)}{3\sigma}) = P(Z < 3C_{pk})$

2) $P(Z < \frac{LSL - \mu}{\sigma}) = P(Z < \frac{USL - USL + LSL - \mu}{\sigma}) =$

$= P(Z < \frac{USL - \mu - (USL - LSL)}{\sigma}) =$

$= P(Z < \frac{3(USL - \mu)}{3\sigma} - \frac{6(USL - LSL)}{6\sigma}) = P(Z < 3C_{pk} - 6C_p)$

$C_p = 1.67$ $\quad\quad$ $C_{pk} = 1.33$

$p = 1 - P(Z < 3C_{pk}) + P(Z < 3C_{pk} - 6C_p) =$

$= 1 - P(Z < 3 \cdot 1.33) + P(Z < 3 \cdot 1.33 - 6 \cdot 1.67) =$

$= 1 - P(Z< 3.99) + P(Z<-6.03) = 1 - 0.999967 + 1 - P(Z< 6.03) =$

$= 0.000033$

b) If the process is out-of-control then the value of p is not reliable.