Tommy Norberg
December 2, 2011

HA5) Multivariate SPC

1. A product has two quality characteristics x_1, x_2. It is known with high confidence that $x_1, x_2 \sim \mathrm{N}(10, 15; 2, 3, 0.6)$, i.e,

$$Ex_1 = 10 \quad Ex_2 = 15$$

$$\mathrm{Var}\, x_1 = 2^2 \quad \mathrm{Var}\, x_2 = 3^2 \quad \text{and} \quad \rho = 0.6$$

   Construct a Hotelling T^2 control chart for monitoring the mean vector $[\bar{x}_1 \; \bar{x}_2]^t$, for the sample size $n = 5$ and the significance level $\alpha = 0.01$. Assume (a) $\bar{x}_1 = 11.2$, $\bar{x}_2 = 13.5$, and (b) $\bar{x}_1 = 11.5$, $\bar{x}_2 = 13.0$. For both cases, answer the following two questions: Is the process in- or out-of-control? In the latter case, which characteristic, if any, seems to be the cause? (Answer without looking at the normalized distances between the sample means and their expectations.)

2. (Continuation.) Assume now that the means and covariances are unknown. The Phase 1 operation consisted consisted of 30 samples of size 5, and resulted in the following estimates

$$\bar{x} = \begin{bmatrix} 10.35 \\ 15.62 \end{bmatrix} \text{ and } S = \begin{bmatrix} 5.72 & 4.85 \\ 4.85 & 7.13 \end{bmatrix}$$

   Construct a Hotelling T^2 control chart for monitoring the mean vector $[\bar{x}_1 \; \bar{x}_2]^t$, for the sample size $n = 5$ and the significance level $\alpha = 0.01$. Answer the same two questions for (a) and (b) as in Problem 1 above.

3. (Continuation.) Is the parameter values used in the T^2-chart in Problem 1 reasonable in view of the Phase 1 estimates in Problem 2? Discuss.

Deadline for solutions to this assignment is Friday, December 9, 2011.