

Sample covariance $S_{xy} = \frac{1}{n-1} \sum_{i=1}^n (X_i - \bar{X})(Y_i - \bar{Y})$ is an unbiased estimate of the population covariance $Cov(X, Y) = E(XY) - E(X)E(Y)$.

Proof. The equality

$$\begin{aligned}
 \sum_{i=1}^n (X_i - \bar{X})(Y_i - \bar{Y}) &= \sum (X_i Y_i) - \bar{X} \sum (Y_i) - \bar{Y} \sum (X_i) + n(\bar{X}\bar{Y}) \\
 &= \sum (X_i Y_i) - n(\bar{X}\bar{Y}) \\
 &= \sum (X_i Y_i) - n^{-1} \left(\sum_{i=1}^n X_i \right) \left(\sum_{j=1}^n Y_j \right) \\
 &= \sum (X_i Y_i) - n^{-1} \left(\sum_{i=1}^n X_i Y_i \right) - n^{-1} \left(\sum_{i \neq j} X_i Y_j \right)
 \end{aligned}$$

entails due to independence between X_i and Y_j that

$$\begin{aligned}
 E\left(\sum_{i=1}^n (X_i - \bar{X})(Y_i - \bar{Y})\right) &= n(1 - n^{-1})E(XY) - n(n-1)n^{-1}E(X)E(Y) \\
 &= (n-1)Cov(X, Y).
 \end{aligned}$$