Tentamentsskrivning i Statistisk slutledning MVE155/MSG200/MSA840/MVE060, 7.5 hp.

Tid: Fredagen den 12 mars, 2010 kl 08.30-12.30
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Hjälpmedel: Chalmersgodkänd räknare, **egen** formelsamling (4 sidor på 2 blad A4) samt utdelade tabeller.
CTH: för "3" fordras 12 poäng, för "4" - 18 poäng, för "5" - 24 poäng.
GU: för "G" fordras 12 poäng, för "VG" - 20 poäng.

Important! For each problem
- describe and justify statistical models you apply,
- state clearly hypotheses you test,
- discuss different relevant approaches you have learned in the course.

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1. (5 points) The data in the accompanying table resulted from an experiment run in a completely randomized design in which each of four treatments was replicated five times.

						Total	Mean
Group 1	6.9	5.4	5.8	4.6	4.0	26.70	5.34
Group 2	8.3	6.8	7.8	9.2	6.5	38.60	7.72
Group 3	8.0	10.5	8.1	6.9	9.3	42.80	8.56
Group 4	5.8	3.8	6.1	5.6	6.2	27.50	5.50
All Groups						135.60	6.78

The next table summarizes the data in terms of the sums of squares, the degrees of freedom and the mean sum squares:

Source	SS	DF	MS
Treatments	38.820	3	12.940
Error	21.292	16	1.331

a. State the null hypothesis of interest. Would you reject it? Describe the statistical model that you use here.

b. Using the percentile of the studentized range distribution $q_{4,16}(0.05) = 4.05$ find out which of the 6 pairwise comparisons among four means $\mu_1, \mu_2, \mu_3, \mu_4$ give significant results.

c. Explain in few words the problem of multiple comparison or multiple testing.

2. (5 points) A school system has around 20,000 third graders, 55% boys and 45% girls. The reading test results from previous years reveal consistently higher variation among the boys' scores as compared to the girls' scores. Namely, the variance of the boys' scores is estimated to be 2.4 times larger than the variance of the girls' scores.

This year, to estimate the overall mean the researchers plan to use a stratified sample of 36 third graders, with one stratum consisting of boys and the other, girls. How many sampled students should be boys and how many should be girls?

3. (5 points) Does heavy exercise increase the risk of myocardial infarction? Mittleman et al. (1993) studied this question by examining the activities of 1228 patients who had suffered myocardial infarctions. It was determined whether each patient had participated in heavy exertion in the hour before the onset of the infarction and also whether each had participated in heavy exertion at the same time the previous day. Their results are displayed in the following table:

	Exertion / infarction day	No exertion / infarction day	Total
Exertion / previous day	4	9	13
No exertion / previous day	50	1165	1215
Total	54	1174	1228

Does the study demonstrate that heavy exertion is associated with myocardial infarction?

4. (5 points) The concentrations (in nanograms per milimiter) of plasma epinephrine were measured for 10 dogs under isofluorane, halothane, and cyclopropane anesthesia; the measurements are given in the following table (Perry et al. 1974).

Dogs	1	2	3	4	5	6	7	8	9	10
Isofluorane	.28	.51	1.00	.39	.29	.36	.32	.69	.17	.33
Halothane	.30	.39	.63	.68	.38	.21	.88	.39	.51	.32
Cyclopropane	1.07	1.35	.69	.28	1.24	1.53	.49	.56	1.02	.30

Use a nonparametric test to find if there is a difference in treatment effects.

5. (5 points) An engineer wants to estimate the probability θ that for a certain type of measurement a given threshold will be exceeded. In 10 such measurement experiments the threshold value was exceeded 4 times.

a. Suppose the engineer had no prior information about the value of θ. Suggest a posterior distribution for θ. Find the posterior mean estimate.

b. Using normal approximation compute a 95% credibility interval for θ.

c. What is the difference in interpretation of a confidence interval and a credibility interval?

6. (5 points) The following inequalities

$$(x^{-1} - x^{-3})e^{-x^2/2} \leq \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-u^2/2} du \leq x^{-1} e^{-x^2/2}$$

hold for any $x > 0$ and give accurate bounds for the normal distribution tails.

a. If Z is normally distributed with mean 2 and variance 4 what is the left tail probability $P(Z < -6)$?

x	1	2	3	4
$e^{-x^2/2}$	0.6065	0.1353	0.0111	0.0003

b. If a random variable X has a symmetric distribution with mean μ and variance σ^2, then its tails are considered to be heavier than normal, if the tail probabilities $P(X > \mu + \sigma x)$ are noticeably exceed $x^{-1}e^{-x^2/2}$ for larger values of x. Sketch a normal probability plot reflecting such a case. Explain why it does look this way.

c. Why a heavy-tailed distribution should be classified as leptokurtic? (A leptokurtic distribution has a more acute peak around the mean as compared to the normal density curve.)

Statistical tables supplied:
1. Normal distribution table
2. Chi-square distribution table
3. t-distribution table
4. F-distribution table

Partial answers and solutions are also welcome. Good luck!

NUMERICAL ANSWERS

1a. One-way ANOVA. The observed F-statistics is $\frac{MS_A}{MS_E} = 9.72$. According to the F-distribution table $F_{.99}(3,16) = 5.29$. Thus, the P-value of the test is less than 1% and we can reject the null hypothesis of no difference among four treatments.

1b. Tukey's simultanuous confidence intervals

$$(\bar{Y}_{u.} - \bar{Y}_{v.}) \pm 4.05\frac{\sqrt{1.331}}{\sqrt{5}} = (\bar{Y}_{u.} - \bar{Y}_{v.}) \pm 2.09.$$

We conclude that four pairs of treatments are significantly different: (1,2), (1,3), (2,4), and (3,4).

2. Stratified sampling, optimal allocation formula. Number of boys

$$n_1 = n \cdot \frac{W_1\sigma_1}{\bar{\sigma}} = 36 \cdot \frac{0.55\sqrt{2.4\sigma_2^2}}{0.55\sqrt{2.4\sigma_2^2} + 0.45\sigma_2} = 36 \cdot \frac{0.55\sqrt{2.4}}{0.55\sqrt{2.4} + 0.45} \approx 24,$$

and the number of girls $n_2 = n - n_1 = 12$.

3. Matched pairs design. McNemara's test statistics is $\frac{(9-50)^2}{9+50} = 28.5$. The null distribution is approximated by the χ_1^2-distribution. Since the square root of 28.5 is larger than 5, the standard normal distribution gives a P-value smaller than 0.0001 (see problem 6). Thus the data clearly demonstrates association between heavy exertion and myocardial infarction.

4. Two-way layout. Friedman's test: 10 dogs judge 3 treatments

Dogs	1	2	3	4	5	6	7	8	9	10	Total ranks
Isofluorane	1	2	3	2	1	2	1	3	1	3	19
Halothane	2	1	1	3	2	1	3	1	2	2	18
Cyclopropane	3	3	2	1	3	3	2	2	3	1	23

The test statistics $Q = 1.4$ is too small to reject the null hypothesis of no difference among three treatments. The approximate null distribution is the χ_2^2-distribution, whose table gives the P-value larger than 10% ($1.4 < 4.61$).

5a. Using the uniform Beta(1,1) prior we arrive at the posterior distribution Beta(1+4,1+10-4)= Beta(5,7). The posterior mean estimate for θ becomes $\hat{\theta}_{pme} = \frac{5}{5+7} = 0.42$.

5b. The variance of the Beta(5,7)-distribution is $\frac{0.42(1-0.42)}{5+7+1} = 0.019$. The approximate 95% credibility interval is $0.42 \pm 1.96\sqrt{0.019} = (0.15, 0.68)$.

6a. Writing $\Phi(x)$ for the cumulative distribution function for the standard normal distribution we get

$$P(Z < -6) = P\left(\frac{Z-2}{\sqrt{4}} < \frac{-6-2}{\sqrt{4}}\right) = \Phi(-4) = \Phi(4) \approx \frac{0.0003}{4} \approx 0.00008.$$