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Solutions chapter 13

Warning: in some of the contingency tables the expected counts are rounded. If you then will compute the chi-square test statistic X^2 from the table, you will often get a somewhat different value.

Problem 13.1

Test

H_0: same genotype frequencies for diabetics and normal

using the chi-square test of homogeneity.

	Diabetic	Normal	Total
Bb or bb	12 (7.85)	4 (8.15)	16
BB	39 (43.15)	49 (44.85)	88
Total	51	53	104

Observed X^2=5.10, df=1, P-value $P = 0.024$. Reject H_0. Diabetics have genotype BB less often.

The exact Fisher test uses Hg(104,51,$\frac{16}{104}$) as the null distribution of the test statistic $N_{11} = 12$

one-sided P-value: 1-hygecdf(11,104,16,51)=0.0225,
two-sided P-value $P = 0.045$.

Normal approximation of the null distribution

$$\text{Hg}(104, 51, \tfrac{16}{104}) \approx \text{ N}(7.85, 3.41).$$

Since $Z = \frac{12-7.85}{\sqrt{3.41}}$=2.245, the approximate two-sided P-value $P = 0.025$.

Problem 13.3

Incidence of tuberculosis in relation to blood groups.

(a) H_0: no association of the disease and the ABO blood group:

	O	A	AB	B	Total
Moderate	7 (10.4)	5 (9.8)	3 (2.0)	13 (6.2)	28
Minimal	27 (30.4)	32 (29.7)	8 (6.1)	18 (18.8)	85
Not present	55 (48.6)	50 (47.5)	7 (9.8)	24 (30.0)	136
Total	89	87	18	55	249

Observed X^2=15.37, df=6, P-value $P = 0.018$. Reject H_0.

(b) H_0: no association of the disease and the MN blood group:

	MM	MN	NN	Total
Moderate	21 (16.7)	6 (9.4)	1 (1.9)	28
Minimal	54 (51.3)	27 (28.9)	5 (5.8)	86
Not present	74 (81.1)	51 (45.7)	11 (9.2)	136
Total	149	84	17	250

Observed X^2=4.73, df=4, P-value $P = 0.42$. Can not reject H_0.

Problem 13.6

Goodness of fit chi-square test for H_0: boys proportions $p_{12} = p_{22} = p_{32} = 0.513$, same sex ratio for three father's activities. (Here 0.513 is obtained as $105.37/(105.37 + 100) = 0.513$.)

	Girl	Boy	Total
Flying fighter	51 (45.15)	38 (43.84)	89
Flying transport	14 (15.22)	16 (14.78)	30
Not flying	38 (42.62)	46 (41.38)	84
Total	103	100	203

Observed X^2=2.75, df=3, P-value $P = 0.43$. Can not reject H_0.

Problem 13.8

A randomized double-blind experiment compared the effectiveness of several drugs in ameliorating postoperative nausea. All patients were anesthetized with nitrous oxide and ether. The following table shows the incidence of nausea during the first four hours for each of several drugs and a placebo.

	Number of patients	Incidence of nausea
Placebo	165	95
Chlorpromazine	152	52
Dimenhydrinate	85	52
Pentobarbital (100 mg)	67	35
Pentobarbital (150 mg)	85	37

We compare the drugs to each other and to placebo using the chi-square test for homogeneity

	No nausea	Incidence of nausea	Total
Placebo	70 (84)	95 (81)	165
Chlorpromazine	100 (78)	52 (74)	152
Dimenhydrinate	33 (43)	52 (42)	85
Pentobarbital (100 mg)	32 (34)	35 (33)	67
Pentobarbital (150 mg)	48 (43)	37 (42)	85
Total (150 mg)	283	271	554

The observed test statistic $X^2 = 35.8$ according to the χ^2_4-distribution table gives P-value $= 3 \cdot 10^{-7}$. Comparing the observed and expected counts we conclude that Placebo and Dimenhydrinate are most effective in ameliorating postoperative nausea.

Problem 13.17

A study of the relation of blood type to peptic ulcer.

(a) H_0: no relation between blood group and disease in London:

	Control	Peptic Ulcer	Total
Group A	4219 (4103.0)	579 (695.0)	4798
Group O	4578 (4694.0)	911 (795.0)	5489
Total	8797	1490	10287

Observed X^2=42.40, df=1, P-value $P = 0.000$. Reject H_0. Odds ratio $\hat{\Delta} = 1.45$.

(b) H_0: no relation between blood group and disease in Manchester:

	Control	Peptic Ulcer	Total
Group A	3775 (3747.2)	246 (273.8)	4021
Group O	4532 (4559.8)	361 (333.2)	4893
Total	8307	607	8914

Observed X^2=5.52, df=1, P-value $P = 0.019$. Reject H_0. Odds ratio $\hat{\Delta} = 1.22$.

(c) H_0: London Group A and Manchester Group A have the same propensity to Peptic Ulcer:

	C and A	PU and A	Total
London	4219 (4349.2)	579 (448.8)	4798
Manchester	3775 (3644.8)	246 (376.2)	4021
Total	7994	825	8819

Observed X^2=91.3, df=1, P-value $P = 0.000$. Reject H_0.

H_0: London Group O and Manchester Group O have the same propensity to Peptic Ulcer:

	C and O	PU and O	Total
London	4578 (4816.5)	911 (672.5)	5489
Manchester	4532 (4293.5)	361 (599.5)	4893
Total	9110	1272	10382

Observed X^2=204.5, df=1, P-value $P = 0.000$. Reject H_0.

Problem 13.18

D = endometrical carcinoma, X = estrogen taken at least 6 months prior to the diagnosis of cancer.

(a) Matched controls, retrospective case-control study

	Controls: estrogen used	Controls: estrogen not used	Total
Cases: estrogen used	39	113	152
Cases: estrogen not used	15	150	165
Total	54	263	317

Apply McNemar test for

$$H_0 : \pi_{1.} = \pi_{.1} \quad \text{vs} \quad H_1 : \pi_{1.} \neq \pi_{.1}.$$

Observed value of the test statistic

$$X^2 = \tfrac{(113-15)^2}{113+15} = 75$$

is highly significant as $\sqrt{75} = 8.7$ and the corresponding two-sided P-value obtained from N(0,1) table is very small.

(b) Possible weak points in a retrospective case-control design

- selection bias: some patients have died prior the study,
- information bias: have to rely on other sources of information.

Problem 13.19

The effect of anxiety on a person's desire to be alone or in company.

	Wait Together	Wait Alone	Total
High-Anxiety	12	5	17
Low-Anxiety	4	9	13
Total	16	14	30

Test

H_0: same frequencies ratio for High-Anxiety and Low-Anxiety groups.
What is a reasonable one-sided alternative?

(a) The exact Fisher test uses Hg(30,17,$\frac{16}{30}$) as the null distribution of the test statistic $N_{11} = 12$. It gives

one-sided P-value: 1-hygecdf$(11, 30, 16, 17) = 0.036$,
two-sided P-value $P = 0.071$.

(b) Using normal approximation

$$\text{Hg}(30, 17, \tfrac{16}{30}) \approx \text{N}(9.1, (1.4)^2)$$

and continuity correction, we find the one-sided P-value to be

$$\text{P}_{H_0}(N_{11} \geq 12) = \text{P}_{H_0}(N_{11} > 11) \approx 1 - \Phi(\tfrac{11.5-9.1}{1.4}) = 1 - \Phi(1.71) = 0.044.$$

(c) Approximate chi-square test yields: observed X^2=4.69, df=1, two-sided P-value

$$2(1 - \Phi(\sqrt{4.69})) = 2(1 - \Phi(2.16)) = 0.03.$$

Problem 13.24

Red against blue outfits - does it matter in combat sports? The winners in different sports

	Red	Biue	Total
Boxing	148	120	268
Freestyle wrestling	27	24	51
Greco-Roman wrestling	25	23	48
Tae Kwon Do	45	35	80
Total	245	202	447

Denote

π_1 = probability that red wins in boxing,
π_2 = probability that red wins in freestyle wrestling,
π_3 = probability that red wins in Greco-Roman wrestling,
π_4 = probability that red wins in Tae Kwon Do.

(a) Assuming

$$H_{eq}: \pi_1 = \pi_2 = \pi_3 = \pi_4 = \pi,$$

we test

$$H_0: \pi = \tfrac{1}{2} \quad \text{vs} \quad H_1: \pi \neq \tfrac{1}{2}.$$

We use the large sample test for proportion based on the statistic $X = 245$ whose null distribution is $\text{Bin}(n, \frac{1}{2})$, $n = 447$. The two-sided P-value is approximated by

$$2(1 - \Phi(\tfrac{245-\frac{447}{2}}{\sqrt{447\cdot\frac{1}{2}}}) = 2(1 - \Phi(2.034) = 0.042.$$

At 5% level we reject the $H_0: \pi = \frac{1}{2}$. The MLE is $\hat{\pi} = \frac{245}{447} = 0.55$.

(d) Is there evidence that wearing red is more favourable in some of the sports than others? We test

$$H_{eq}: \pi_1 = \pi_2 = \pi_3 = \pi_4 \quad \text{vs} \quad H_{ineq}: \pi_i \neq \pi_j \quad \text{for some } i \neq j$$

using the chi-square test of homogeneity. From

	Red	Biue	Total
Boxing	148 (147)	120 (121)	268
Freestyle wrestling	27 (28)	24 (23)	51
Greco-Roman wrestling	25 (26)	23 (22)	48
Tae Kwon Do	45 (44)	35 (36)	80
Total	245	202	447
Marginal proportions	0.55	0.45	1.00

we find that the test statistic $X^2 = 0.3$ is not significant. We can not reject H_{eq}, which according to (a) leads to $\hat{\pi} = 0.55$.

(b) Now we state the hypotheses of interest directly: consider

$$H_0: \pi_1 = \pi_2 = \pi_3 = \pi_4 = \tfrac{1}{2} \quad \text{vs} \quad H_1: (\pi_1, \pi_2, \pi_3, \pi_4) \neq (\tfrac{1}{2}, \tfrac{1}{2}, \tfrac{1}{2}, \tfrac{1}{2}).$$

Here we need a new chi-square test, a chi-square test for k proportions with $k = 4$. We derive it using the likelihood ratio approach.

Given four observed counts $x_1 = 148$, $x_2 = 27$, $x_3 = 25$, $x_4 = 45$, the likelihood function based on the binomial model has the form

$$L(\pi_1, \pi_2, \pi_3, \pi_4) = \prod_{i=1}^{4} \binom{n_i}{x_i} \pi_i^{x_i}(1-\pi_i)^{n_i - x_i}.$$

Using $\hat{\pi}_i = \frac{x_i}{n_i}$, we compute the likelihood ratio as

$$\Lambda = \frac{L(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2})}{L(\hat{\pi}_1, \hat{\pi}_2, \hat{\pi}_3, \hat{\pi}_4)} = \frac{(\frac{1}{2})^n}{\prod_{i=1}^{4} (\frac{x_i}{n_i})^{x_i} (\frac{n_i - x_i}{n_i})^{n_i - x_i}}.$$

Turning to the logarithms,

$$\Delta = -\ln\Lambda = \sum_{i=1}^{4} x_i \ln\tfrac{2x_i}{n_i} + (n_i - x_i)\ln\tfrac{2(n_i - x_i)}{n_i},$$

we take 2Δ as the test statistic.

Next we show that the null distribution of 2Δ is approximately χ^2_4. Under H_0 we have $\frac{2x_i}{n_i} \approx 1$, and using a Taylor expansion we find that

$$2\Delta \approx X^2 = \sum_{i=1}^{4} \tfrac{(x_i - \frac{n_i}{2})^2}{n_i/4},$$

where $Z_i = \frac{X_i - \frac{n_i}{2}}{\sqrt{n_i/4}}$ are independent and approximately N(0,1) distributed, provided $X_i \sim \text{Bin}(n_i, \frac{1}{2})$. From

	Red	Biue	Total
Boxing	148 (134)	120 (134)	268
Freestyle wrestling	27 (25.5)	24 (25.5)	51
Greco-Roman wrestling	25 (24)	23 (24)	48
Tae Kwon Do	45 (40)	35 (40)	80
H_0 proportions	0.5	0.5	1.00

we find $X^2_{\text{obs}} = 4.4$. Since $\chi^2_4(0.1) = 7.8$, we do not reject $H_0: \pi_1 = \pi_2 = \pi_3 = \pi_4 = \frac{1}{2}$.