

# MSG800/MVE170 and MVE172

## List of Errata for Hsu's book, Version 14 November 2020

**Equation 2.41.**  $F_X(X)$  should be  $F_X(x)$ .

**Equation 2.53.** Just after (2.53) it should be  $\lfloor x \rfloor$  instead of  $|x|$ . In 3'rd and 4'th Ed of printed book this needs correction but is corrected in e-version of 4'th Ed.

**Equation 3.14.** It should be  $\lim_{x \rightarrow \infty}$  instead of  $\lim_{y \rightarrow \infty}$ .

**Exercise 3.12.**  $P(Y = 1 | X = 1)$  should be  $P(Y = 0 | X = 1)$  on diagonal in figure.

**Problem 4.100.**  $X''$  should be corrected to  $X^n$ .

**Section 5.4 B.** The definition of WSS should be the conditions (5.21)-(5.22): "Stationary of order 2" is a much stronger condition.

**Equation 5.28** is wrong as is the proof of (another version of) that formula in Problem 5.25: The second equality in the three line equation at the middle of the solution is erroneous. To see this we may send  $x_{n-1} \rightarrow \infty$  on both sides of the equation to obtain

$$F_X(x_1, \dots, x_{n-2}, x_n; t_1, \dots, t_{n-2}, t_n) = F_X(x_n; t_n) \times F_X(x_1, \dots, x_{n-2}; t_1, \dots, t_{n-2}),$$

which is to say that  $X(t_n)$  is independent of  $X(t_1), \dots, X(t_{n-2})$ . This is absurd as this need not hold at all for a Markov process (as is e.g., exemplified by a Poisson process).

A correct version of Equation 5.28 is the formula for PMF's given in Problem 5.88.

**Equation 5.29.** It should be  $E[X(t_i)]$  instead of just  $E[X(t_i)]$ .

**Equation 5.51.** It is in Problem 5.94 that (5.51) is proved - not in Problem 5.74.

**Lemma 5.8.1.** It should be  $T_1$  and  $T_2$  instead of  $n_1$  and  $n_2$ , respectively.

**Optional stopping theorem** is wrongly called optimal stopping theorem in 3'rd printed Ed of book.

**Equation 5.121.** It should be  $\cos \omega \tau$  instead of  $\cos \omega t$  in the middle row.

**Problem 5.37.** It should be  $\gcd\{2, 4, 6, \dots\}$  instead of  $\gcd\{2, 5, 6, \dots\}$ .

**Problem 5.49.** Four occurrences of  $X(t + \Delta t) - X(0)$  should be  $X(t + \Delta t) - X(t)$ .

**Equation 5.202.** It should be  $\sigma$ 's instead of  $a$ 's in the  $\mathbf{K}_X$ -matrix.

**Problem 5.70.** The alternative possible value of  $X_i$  should be  $-1$  with probability  $q = 1 - p$ . In 3'rd Ed of printed book this needs correction but is corrected in both printed and e-version of 4'th Ed. And such a random variable is not called Bernoulli distributed.

**Problem 5.72.** Here  $\frac{n(n-1)}{2}$  should be  $n(n-1)$  and  $\frac{n(n+3)}{2}$  should be  $n(n+1)$ .

**Problem 5.74.** Here  $\frac{n+2+k}{n+2}$  should be  $\frac{n+2-k}{n+2}$ .

**Problem 5.76.** Change  $E(|g(Xn)|)$  to  $E(|g(X_n)|)$ .

**Problem 5.77.** A correct proof that  $\mathbf{E}\{|X_n|\} < \infty$  is as follows:

$$\mathbf{E}\{|X_n|\} = \mathbf{E}\{|\mathbf{E}\{X|F_n\}|\} \leq \mathbf{E}\{\mathbf{E}\{|X||F_n\}\} = \mathbf{E}\{|X|\} < \infty.$$

The initial condition should be  $\mathbf{E}\{|X|\} < \infty$  instead of  $\mathbf{E}\{X\} < \infty$  accordingly.

**Problem 5.78.** In 3'rd and 4'th Ed of printed book Theorem 5.82 should be Theorem 5.8.2 at two occurrences but this is corrected in e-version of 4'th Ed.

**Problem 5.79.** One occurrence of  $\{T > k - 1\}$  should be  $\{T > k - 1\}$ .

**Problem 5.81.** At beginning of solution of problem a one occurrence of  $s_n$  should be  $S_n$  and close to end of solution of problem b the first  $\frac{a}{a+b}$  should be  $\frac{b}{a+b}$ .

**Problem 5.82.** It should be  $X(t) - \lambda t$  instead of  $x(t) - \lambda t$  at two places.

**Problem 6.8 b.** At the beginning of the solution of problem b it should be  $\partial^2 R_X(s - t)/\partial t \partial s = -d^2 R_X(\tau)/d\tau^2$ , i.e., a  $-$  is missing.

**Problem 6.12.** In the solution of problem b one occurrence of  $E[X(s)]X(\beta)$  should be  $E[X(s)X(\beta)]$  and in the equation before (6.124) the  $s^3$  on the right hand side should be  $s^2$ .

**Equation 6.137.** It should be  $R_{X'}(t, s)$  instead of  $R_X(t, s)$ .

**Problem 6.26.** In the right most expression for  $S_Y(\omega)$  factor  $\frac{a}{(a^2+b^2)b}$  should be  $\frac{a}{(a^2-b^2)b}$ .

**Equation 9.20.** In the sum it should be  $(s\rho)^n$  instead of just  $(s\rho)$ .

**Equation 9.22.** Change  $(sp)^s$  to  $(s\rho)^s$ .

**Equation 9.36.** Change  $\rho_0$  to  $p_0$ .

**Equation 9.39.** On the right-hand side  $L_q$  should be divided by  $\lambda_e = \lambda(1 - p_K)$ .

**Problem 9.13.** Beginning of solution to problem a reference to (9.16) should be to (9.17). In the final evaluation of  $W_q$  the term  $-1/\mu = -3$  which should be there according to (9.54) has been forgotten in the middle term (but the answer to the right 6.39 min is correct).

**Problem 9.16.** The "balance equation" (9.2)  $L = \lambda W$  holds for all steady-state queues provided that we set the total time spent in the system to zero for customers arriving when the system is full for queues with  $K < \infty$ , i.e., if we interpret the fact that customers arriving to a full system bounces away as that they spend zero time in system.

If we instead (as is custom and as is done in the book) define  $W$  as the expected total time spent in the system for customers that really join the system, then we must divide the  $W$  from the previous paragraph by  $1 - p_K$  to get this  $W$ . And then we have (9.31)  $L = \lambda(1 - p_K)W$  for this alternative (and customary/correct) definition of  $W$ .

The sum in the second last equation of the solution should only run to  $K - 1$ , giving  $W_q = (L - K p_K)/\mu$ . The sum in the last equation of the solution should also only run to  $K - 1$  giving  $W = (L + 1 - (K + 1)p_K)/\mu$ . Now, here we are dealing with the first definition of  $W$  from above, i.e., we assign zero time in the system for bouncing customers and let that zero influence the mean value  $W$ , so that  $L = \lambda W$  for this  $W$ .

As we want the customary/correct definition of  $W$  to be the expected total time spent in the system for customers that really joins the system we have to divide the  $W$  obtained

in the previous paragraph by  $1 - p_K$  giving the following correct version (9.59):  $W = (L + 1 - (K + 1)p_K)/(\mu(1 - p_K))$ . The corresponding correct version of (9.58) is  $W_q = (L + 1 - (K + 1)p_K)/(\mu(1 - p_K)) - 1/\mu = (L - K p_K)/(\mu(1 - p_K))$ .

**Problem 9.18.** In the middle of the calculation  $p^m$  should be changed to  $\rho^m$ .

**Problem 9.29.** To solve problem a Hsu has used faulty (9.59) - see above - to obtain  $W = (L + 1)/\mu = \dots = 0.336$  hours = 20.15 minutes, while a correct answer comes from using the correct version of that formula from above  $W = (L + 1 - (K + 1)p_K)/(\mu(1 - p_K)) = (L - K p_K)/(\mu(1 - p_K)) + 1/\mu = 7/57 + 1/6 = 0.289$  hours = 17.4 minutes, or from using (9.31). The correct answer to problem b is  $W_q = W - 1/\mu = W - 10$  minutes = 7.4 minutes.

**Problem 9.30.** The correct answer is 1.

**Table B-2.** In Formula 11 the right hand side should be  $X(\omega) = (\pi/a) e^{-a|\omega|}$ .