MSG800/MVE170 Basic Stochastic Processes

Exercise Session 3

Chapter 5 in Hsu's book (continued)

Solved problems. Problems 5.72, 5.76, 5.77, 5.78, 5.81 and 5.82 in Hsu's book.

Problems for own work. Problems 5.104, 5.105 and 5.106 in Hsu's book.

Computer problem for own work. A Nevada own-home-owner bicycles to Las Vegas in order to bet all his money \$100 on a \$1 slot machine hoping to end up doubling his capital to make it possible for him to pay a pending \$200 mortage on his own-home. The slot machine game process is a time discrete stochastic process $M_n = \sum_{i=1}^{n} X_i$ for $n \in \mathbb{N}$, where $M_0 = 100$ is the initial capital and $\{X_i\}_{i=1}^{\infty}$ is a sequence of independent identically distributed random variables with $\mathbf{P}\{X_i = 4\} = 1/5$ and $\mathbf{P}\{X_i = -1\} = 4/5$ describing the gain or loss at the consecutive pulls of the slot machine. The own-home-owner carries on the slot machine game until the first time $T = \min\{n \geq 1 : M_n \geq 200 \text{ or } M_n = 0\}$ at which either the capital is doubled (in which case he bicycles home and pays his mortage) or the capital is depleted (in which case he bicycles home to sell his own-home).

The process $\{M_n\}_{n=0}^{\infty}$ is a martingale and the random time T is a stopping time that satisfy the conditions of the optimal stopping theorem. (It is a very useful exercise for the extra ambitious student to prove that this is really so.) Writing p for the probability that the own-home-owner can pay his mortage it follows from the optional stopping theorem that

$$100 = \mathbf{E}\{M_0\} = \mathbf{E}\{M_T\} \begin{cases} \leq p \cdot 203 \\ \geq p \cdot 200 \end{cases},$$

so that $p \in [\frac{100}{203}, 1/2]$. Find by means of computer simulations a better estimate of the value of the probability p than that obtained in this manner from the optional stopping theorem.

Chapter 6 in Hsu's book

Solved problems. Problems 6.7, 6.8, 6.9 and 6.12 in Hsu's book.

Supplementary problems for own work. Problems 6.47, 6.49 and 6.50 in Hsu's book.