MSG800/MVE170 Basic Stochastic Processes

Written home re-exam Tuesday 24 August 2021 2–6 PM

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Aids: All aids are permitted. (See the Canvas course "Omtentamen 2 Modul: 0107, MVE170" with instructions for this reexam for clarifications.)

Grades: 12 points for grades 3 and G, 18 points for grade 4, 21 points for grade VG and 24 points for grade 5, respectively.

Motivations: All answers/solutions must be motivated. Good Luck!

Task 1. Find a WSS continuous time random process $\{X(t)\}_{t\in\mathbb{R}}$ that is not strict sense stationary. **(5 points)**

Task 2. Calculate $\Pr(X(0)=0)$ for a zero-mean WSS random process with autocorrelation function $R_{XX}(\tau)=0$. **(5 points)**

Task 3. Calculate $\Pr(X(1)X(2)X(3)=6)$ for a Poisson process $\{X(t)\}_{t\geq 0}$ with arrival rate 1. **(5 points)**

Task 4. A discrete time Markov chain has four states $\{0,1,2,3\}$ and all transition probabilities $p_{ij}=1/4$. Calculate the expected value of the time it takes for the chain to move from state 0 to state 3. **(5 points)**

Task 5. A continuous time Markov chain has four states $\{0,1,2,3\}$ and all off diagonal generator elements $g_{ij}=1/3$ for $i\neq j$. Describe the probability law of the time it takes for the chain to move from state 0 to state 3. **(5 points)**

Task 6. An AR(1)-process $\{X_k\}_{k=-\infty}^{\infty}$ satisfies $X_{k+1}=a\,X_k+\sigma\,e_{k+1}$ where $|a|<1$ and $\sigma>0$ are constants and $\{e_k\}_{k=-\infty}^{\infty}$ is discrete time white noise with e_{k+1} independent of $\{X_\ell\}_{\ell=-\infty}^{k}$. Explain why an AR(1)-process is a Markov chain. **(5 points)**

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Task 1. Let all values of $X(t)$ be independent of each other with zero-mean and unit variance but not all CDF's $F_{X(t)}(x)$ being the same.

Task 2. 1.

Task 3. Clearly $\Pr(X(1)X(2)X(3) = 6) = \Pr(X(1) = 1,\, X(2) = 1,\, X(3) = 6) + \Pr(X(1) = 1,\, X(2) = 2,\, X(3) = 3) = \Pr(X(1) = 1,\, X(2) - X(1) = 0,\, X(3) - X(2) = 5) + \Pr(X(1) = 1,\, X(2) - X(1) = 1,\, X(3) - X(2) = 1) = \Pr(X(1) = 1)\Pr(X(2) - X(1) = 0)\Pr(X(3) - X(2) = 5) + \Pr(X(1) = 1)\Pr(X(2) - X(1) = 1)\Pr(X(3) - X(2) = 1) = \frac{1}{5!}(\mathrm{e}^{-1})^3 + (\mathrm{e}^{-1})^3 = \frac{121}{120}\,\mathrm{e}^{-3}$.

Task 4. For the sought after expectation E we have $E = 1 + (3/4) \cdot E$ giving $E = 4$.

Task 5. As $\Psi_{T_{0\to3}}(\omega) = \frac{1}{1-j\omega}\left(\frac{1}{3} \cdot 1 + \frac{2}{3} \cdot \Psi_{T_{0\to3}}\right)$ we get $\Psi_{T_{0\to3}}(\omega) = \frac{1/3}{1/3-j\omega}$, which is to say that the time is exponentially distributed with mean 3.

Task 6. Because the next value of the process $X_{k+1} = a\,X_k + e_{k+1}$ depends on the history of the process $\{X_\ell\}_{\ell=-\infty}^{k}$ through the last member of the history X_k only (as e_{k+1} is independent of that history).