MVE171 Basic Stochastic Processes and Financial Applications, Exercise Session 0

This exercise session is intended for students who feel they have week skills in probability theory to strengthen those skills.

Chapters 1-2 in Hsu's book

Students who feel insecure about topics in Chapters 1 and 2 in Hsu's book should look for corresponding solved problems therein, as well as possibly a few supplementary problems for own work.

Chapter 3 in Hsu's book

Solved problems. Problems 3.20, 3.30, 3.34 and 3.40 in Hsu's book.

Problems for own work. Problems 3.57 and 3.68 in Hsu's book.

Computer problem for own work. Calculate the integral $\int_0^1 (\sin(1/x))^2\,dx$ numerically by means of the so called Monte-Carlo method, which is to say, generate a very great number n (=as many as you can) independent bivariate random variables $\{(X_i, Y_i)\}_{i=1}^n$ that all have a common unifom distribution over the unit square with PDF $f_{X,Y}(x, y) = 1$ for $0 \leq x, y \leq 1$ and $f_{X,Y}(x, y) = 0$ elsewhere and check how great a fraction of these random numbers that satisfy $(\sin(1/X_i))^2 \geq Y_i$. Also dicuss (theoretically and/or heuristically) why this method should give the correct value for the integral as $n \to \infty$.

The error-term for the Monte-Carlo numerical integration method is of the order $1/\sqrt{n}$ independently of what function that is integrated. Therefore the method is suitable for ill-behaved irregular functions [like, e.g., $(\sin(1/x))^2$] for which classical deterministic numerical integration procedures can be expected to perform poorly.

Chapter 4 in Hsu's book

Equations (equation numbers) and problems (problem numbers) in Chapter 4 of the Second Edition of Hsu's book do not agree completely with those in the First Edition: The main difference is that the Second Edition contains a page or so of additional theoretical material concerning Cauchy-Schwarz inequality, Jensen's inequality and probability generating functions as compared with the First Edition that has been inserted between the section concerning expectations and that concerning moment generating functions. As a result of that additional corresponding problems (exerercises) also have been added to the Second Edition.

A few of the exercises below concern the so called moment generating function (MGF) of a random variable X given by $M_X(t) = E(\mathrm{e}^{tX})$ for $t \in \mathbb{R}$. It is easy to see that $M_X^{(n)}(0) = E(X^n)$ (compare with the corresponding result for characteristic functions), so that $M_X(t) = \sum_{n=0}^{\infty} t^n M_X^{(n)}(0)/(n!)$ by Taylor-expansion (power-series expansion).

Solved problems. Problems 4.17, 4.18, 4.25, 4.45, 4.49, 4.58, 4.59 and 4.77 in the Second Edition of Hsu's book (Problems 4.15, 4.16, 4.21, 4.39, 4.43, 4.47, 4.54 and 4.59 in the First Edition).

Supplementary problems for own work. Problems 4.90, 4.96 and 4.100 in the Second Edition of Hsu's book (Problems 4.69, 4.76 and 4.80 in the First Edition).