MVE171 Basic Stochastic Processes and Financial Applications, Exercise Session 2

Chapter 5 in Hsu's book (continued)

Solved problems. Problems 5.30, 5.32, 5.35, 5.49, 5.55 and 5.60 in Hsu's book.

Problems for own work. Problems 5.92, 5.93, 5.95, 5.98, 5.100 and 5.101 in Hsu's book.

Computer problem for own work. Let $\{W(t)\}_{t\geq 0}$ be a Wiener process with $\sigma^2 = \mathbf{Var}\{W(1)\} = 1$. For a real constant $\varepsilon > 0$, consider the differential ratio process $\Delta_\varepsilon = \{\Delta_\varepsilon(t)\}_{t>0}$ given by

$$\Delta_\varepsilon(t) = \frac{W(t+\varepsilon) - W(t)}{\varepsilon} \quad \text{for } t > 0.$$

For $s > 0$ and $t \in \mathbb{R}$ (the latter of which has an absoulte value small enough to make $s+t \geq 0$), show that the autocorrelation function

$$R_{\Delta_\varepsilon}(t) = R_{\Delta_\varepsilon}(s, s+t) = \mathbf{E}\{\Delta_\varepsilon(s)\Delta_\varepsilon(s+t)\}$$

of Δ_ε is a triangle like function that depends on the difference t between $s > 0$ and $s+t \geq 0$ only. Further, show that $R_{\Delta_\varepsilon}(t) \to \delta(t)$ (Dirac's δ-function) as $\varepsilon \downarrow 0$. Simulate a sample path of $\{\Delta_\varepsilon(t)\}_{t\in(0,1]}$ for a really small $\varepsilon > 0$ (recall that W has independent increnments) and plot a graph of that sample path. Discuss the claim that the (non-existing in the usual sense) derivative process $\{W'(t)\}_{t\geq 0}$ of W is white noise.