MVE171 Basic Stochastic Processes and Financial Applications, Written exam Monday 8 January 2018 $2-5$ pm

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AIDS: Either two A4-sheets (4 pages) of hand-written notes (xerox-copies and/or computer print-outs are not allowed) or Beta (but not both these aids).

GRADES: 8 points for grade 3, 12 points for grade 4 and 16 points for grade 5.

MOTIVATIONS: All answers/solutions must be motivated. GOOD LUCK!

Task 1. Let $\{W(n)\}_{n\in\mathbb{Z}}$ be a discrete time white noise process with $\mathbf{E}\{W(n)^2\}=1$. Find the autocorrelation function $R_X(k,l)$ of the process $X(k)=W(k)+W(k-1)+W(k-2)$. **(5 points)**

Task 2. Give four examples of discrete time random processes where the first one is neither a Markov chain or a martingale, the second a Markov chain but not a martingale, the third a martingale but not a Markov chain and the fourth both a Markov chain and a martingale. **(5 points)**

Task 3. Calculate $\mathbf{P}\{X(0)+X(1)\geq Y(1)+Y(2)+1\}$ when $X(t)$ and $Y(t)$ are independent zero-mean continuous time WSS Gaussian processes with autocorrelation functions $R_X(\tau)=\mathrm{e}^{-|\tau|}$ and $R_Y(\tau)=1/(1+\tau^2)$. **(5 points)**

Task 4. For a continuous time LTI system with WSS input and output $X(t)$ and $Y(t)$, respectively, with PSD's $S_X(\omega)$ and $S_Y(\omega)$, respectively, and with frequency response $H(\omega)$ it holds that $S_Y(\omega)=|H(\omega)|^2S_X(\omega)$. Prove this fact. **(5 points)**

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Task 2. It is easy to see that $X(k)$ is WSS with $R_X(k) = R_X(l, k+l) = 3\,\delta(k) + 2\,\delta(k-1) + 2\,\delta(k+1) + \delta(k-2) + \delta(k+2)$.

Task 2. Take $W(k)$ as in Task 2 and let $X(0) = W(0)$, $X(1) = X(0) + W(1)$ and $X(n) = X(n-1) + X(n-2) + W(n)$ for $n \geq 2$. Then $X(k)$ is a martingale but not Markov while $Y(k) = X(k) + k$ i neither martingale or Markov. The process $Z(k) = 0$ is both martingale and Markov while $Z(k) + k$ is Markov but not martingale.

Task 3. The probability is $1 - \Phi(1/\sigma)$ where $\sigma^2 = \mathbf{Var}\{X(0)+X(1)-Y(1)-Y(2)\} = \mathbf{Var}\{X(0)+X(1)\} + \mathbf{Var}\{Y(1)+Y(2)\} = 2\,(R_X(0)+R_X(1)) + 2\,(R_Y(0)+R_Y(1))$.

Task 4. See Chapter 6 in Hsu's book.