MVE171 Basic Stochastic Processes and Financial Applications

Written exam 27 Tuesday August 2019 2–5 PM

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AIDS: Either two A4-sheets (4 pages) of hand-written notes (xerox-copies and/or computer print-outs are not allowed) or Beta (but not both these aids).

GRADES: 8 points for grade 3, 12 points for grade 4 and 16 points for grade 5.

MOTIVATIONS: All answers/solutions must be motivated. GOOD LUCK!

Task 1. Write a computer programme in some easy to understand code or pseudo-code that determines by means of simulations an approximative value for the probability $\mathbf{P}\{X(t) > t \text{ for some } t \in [0,10]\}$ for a unit rate Poisson process $\{X(t)\}_{t\geq 0}$.

(5 points)

Task 2. Let $Y_n = ((1-p)/p)^{S_n}$ for $n \geq 0$, where $S_0 = 0$, $S_n = \sum_{i=1}^n X_i$ and $X_1, X_2, \ldots$ are independent identically distributed r.v.'s such that $P(X_i = -1) = 1-p$ and $P(X_i = 1) = p$ where $0 < p < 1$ is a constant. Show that $\{Y_n\}_{n=0}^{\infty}$ is a martingale with respect to the filtration $F_n = \sigma(X_1, \ldots, X_n)$. **(5 points)**

Task 3. Give an example of a discrete-parameter Markov chain $\{X(n),\, n \geq 0\}$ that has both periodic and aperiodic states. **(5 points)**

Task 4. Let $\{X(t),\, t \in \mathbb{R}\}$ be a WSS process and $\alpha > 0$ a constant. Show that $\{X(\alpha t),\, t \in \mathbb{R}\}$, $\{X(t-\alpha),\, t \in \mathbb{R}\}$ and $\{X(-t),\, t \in \mathbb{R}\}$ are also WSS processes. **(5 points)**

MVE171 Solutions to written exam 27 August 2019

Task 1.

```
rep=100000;For[i=1;count=0,i<=rep,i++,X=0;succ=0;
                     t=Random[ExponentialDistribution[1]];
While[(t<=10)&&(succ=0),t=t+Random[ExponentialDistribution[1]];
X=X+1;If[X>t,succ=1]];count=count+succ];N[count/rep]
```

Task 2. $\mathbf{E}\{Y_{n+1}|F_n\} = Y_n\,\mathbf{E}\{((1-p)/p)^{X_{n+1}}\} = Y_n\,[((1-p)/p)\cdot p+(p/(1-p))\cdot(1-p)] = Y_n$.

Task 3. For example the chain with state space E and transition probability matrix P given by

$$E = \{0,1,2,3\} \qquad \text{and} \qquad P = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1/2 & 1/2 \\ 0 & 0 & 1/2 & 1/2 \end{bmatrix},$$

respectively, where the states $\{0,1\}$ have period 2 while the states $\{2,3\}$ are aperiodic.

Task 4. Clearly, $X(\alpha t)$, $X(t-\alpha)$ and $X(-t)$ all have the same constant (time-invariant) mean μ_X as has $X(t)$ while $\mathbf{E}\{X(\alpha t)X(\alpha(t+\tau))\} = R_X(\alpha\tau)$, $\mathbf{E}\{X(t-\alpha)X(t+\tau-\alpha)\} = R_X(\tau)$ and $\mathbf{E}\{X(-t)X(-(t+\tau))\} = R_X(-\tau) = R_X(\tau)$ do not depend on $t \in \mathbb{R}$.