MVE172 Basic Stochastic Processes and Financial Applications

Written home re-exam Tuesday 24 August 2021 2–5 PM

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AIDS: All aids are permitted. (See the Canvas course "Omtentamen 2 Modul: 0220, MVE172" with instructions for this reexam for clarifications.)

GRADES: 8, 12 and 16 points for grades 3, 4 and 5, respectively.

MOTIVATIONS: All answers/solutions must be motivated. GOOD LUCK!

Task 1. Find a WSS continuous time random process $\{X(t)\}_{t\in\mathbb{R}}$ that is not strict sense stationary. **(5 points)**

Task 2. Calculate $\Pr(X(0)=0)$ for a zero-mean WSS random process with autocorrelation function $R_{XX}(\tau)=0$. **(5 points)**

Task 3. Calculate $\Pr(X(1)X(2)X(3)=6)$ for a Poisson process $\{X(t)\}_{t\geq 0}$ with arrival rate 1. **(5 points)**

Task 4. A discrete time Markov chain has four states $\{0,1,2,3\}$ and all transition probabilities $p_{ij}=1/4$. Calculate the expected value of the time it takes for the chain to move from state 0 to state 3. **(5 points)**

MVE172 Solutions to written re-exam 24 August 2021

Task 1. Let all values of $X(t)$ be independent of each other with zero-mean and unit variance but not all CDF's $F_{X(t)}(x)$ being the same.

Task 2. 1.

Task 3. Clearly $\Pr(X(1)X(2)X(3) = 6) = \Pr(X(1) = 1,\, X(2) = 1,\, X(3) = 6) + \Pr(X(1) = 1,\, X(2) = 2,\, X(3) = 3) = \Pr(X(1) = 1,\, X(2) - X(1) = 0,\, X(3) - X(2) = 5) + \Pr(X(1) = 1,\, X(2) - X(1) = 1,\, X(3) - X(2) = 1) = \Pr(X(1) = 1)\Pr(X(2) - X(1) = 0)\Pr(X(3) - X(2) = 5) + \Pr(X(1) = 1)\Pr(X(2) - X(1) = 1)\Pr(X(3) - X(2) = 1) = \frac{1}{5!}(\mathrm{e}^{-1})^3 + (\mathrm{e}^{-1})^3 = \frac{121}{120}\mathrm{e}^{-3}$.

Task 4. For the sought after expectation E we have $E = 1 + (3/4) \cdot E$ giving $E = 4$.