MSA101/MVE187 2017 Lecture 14

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Using improper priors

The example from assignment 2.

Model selection and model checking

- Bayesian inference is conceptually simple: Once you have fixed the model (including the prior), the way to proceed is dictated by mathematics.
- But, given data and a context, how to you find the model? This is the most difficult question in (Bayesian) statistics.

Comparing models via Bayes factors

- A set of possible models for the data is selected; each uses a *proper* prior distribution.
- The likelihood of data given each model (i.e., the value of the prior predictive) is computed.
- Ratios of likelihoods of models are called *Bayes factors*.
- If prior probability weights are assigned to each model, posterior probability weights can be computed.
- When only two models: The Bayes factor for a model is the ratio of the posterior odds to the prior odds of this model.
- Example: Albert sections 8.3, 8.4.
- Example: Robert Casella 6.8.

Difficulties

- Instead of determining prior weights for the models, one may compare the model likelihoods: If one is "sufficiently big", one may decide to go with only this model. (A practical alternative to using Hypothesis Testing for model selection).
- Alternatively, one may go on with a weighted mean of the models, but then actual prior weights must be determined. May be particularly difficult to do when the models are structurally different.
- Improper priors may cause difficulties in the setup above.
- Improper priors should not be replaced with "vague" priors for model comparison purposes!
- Main problem: You have to first come up with the list of "possible" models, before you can do model selection using Bayes factors!

Comparing models in other ways?

- In applied statistics, a statistical model is often "validated" based on how "well" it does when applied to case data.
- A version of this is *cross validation*: The data is subdivided into n parts, and for $i = 1, \ldots, n$, all data except the n'th part is used to fit the model, and the likelihood of the n'th part in this fitted model is recorded. E.g., the product of all such likelihoods is maximized.
- Often, such procedures become similar to maximizing the likelihood of the full data under the prior model.

Informal model checking: Simulation from the prior predictive

- The prior model should represent "prior knowedge": A way to check that it does this correctly is to simulate new data from the prior predictive and check if they look like what you expect a priori.
- Examples
  - Simulate from the prior of a stochastic model for tree growth.
  - Simulate from the prior of a stochastic model for geological faults.
  - Simulate from the prior of a stochastic model for image noise.

Informal model checking: Hypothesis testing

- A practical problem with model comparison via Bayes factors is that both (or all) models need to be completely specified.
- Hypothesis testing lets you compare a model with an alternative that deviates from it in the direction measured by the test statistic, but may otherwise be unspecified.
- Thus, hypothesis testing can be used in Bayesian statistics as a way to indicate alternative models.
- Over-interpretation of p-values must be avoided.

Informal model checking: Simulating from the posterior predictive

- The prior will indicate that some "features" of the model can be "informed" by the data, while other "features" are fixed. Are there "features" that are fixed that need to be informed by the data? This can be investigated by comparing simulations from the posterior predictive with the actual data. Are there systematic differences?
- Very simple example:
  - Data, 4.33, 4.32, 4.35, 4.30.
  - Model: $y_i \sim \text{Normal}(\mu, \sigma^2)$.
  - If the prior is $\mu \sim \text{Normal}(0, 100)$, $\sigma^2 = 1$, simulations from the posterior predictive will have too much spread in the data.
  - If the prior is $\mu = 0$, $\pi(\sigma^2) \propto 1/\sigma^2$, simulations from the posterior predictive will hav both wrong mean and wrong spread.
- Posterior predictive p-values
- Heart transplant example in chapter 7 of Albert.

Innformal model checking: Exploratory data analysis

- Any statistical analysis should begin with an exploratory data analysis.
- It is advantageous to view this data exploration as a sequence of informal model checks and model comparisons.

Comparing models based on their complexity

- In model comparisons, you need to weigh how well the models fit the data agains how much prior belief you have in each model.
- But what if one model is much more complex than another, how should that influence prior belief?
- In particular, one model that is a generalization of another will always fit the data better.
- Use of *information criteria* that *penalize* the complexity of a model:
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    - AIC, Akaike Information Criterion.
    - BIC Bayesian Information Criterion.
    - DIC Deviance Information Criterion.
    - ...

Model selection: Learning graphical network models from data

- Given a set of observations of a set of variables, one may assume this is a (random) sample from a joint distribution, and one may try to learn a reasonable set of conditional independencies from the data.
- More concretely, an algorithm produces one (or several) graphical models from the data.
- In principle, the same issues as for Bayesian model selection, or any other model selection, apply.
- Problem: There are a huge number of possible graphs for a moderately long list of variables.
- Important field of research.