MSA101/MVE187 2018 Lecture 12

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Overview

- An example (on blackboard) using Gaussian Markov Random Fields.
- Causal networks
- Exact inference for graphical models.
- Example: The Forward-Backward algorithm.

Causal networks

- Bayesian networks, like stochastic models in general, have nothing to do with causality. ("Correlation is not causation").
- However, one may *add* the following interpretation to a Bayesian network, to obtain a causal network: If one *intervenes* at a node (which is different from *observing* the value of the node) the probability distribution for the remaining nodes is given by the Bayesian network obtained from the old one by removing the conditional distribution for the intervention node.
- Example, with rain and umbrella.
- In general, one would like to infer causal networks from data: Methods may be difficult and controversial.

Computations for Graphical models

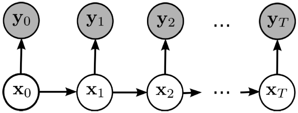
- Given a graphical model, we might want to
  - Follow the Bayesian paradigm and find the conditional distribution of some nodes (variables) given fixed values (data) for some other nodes. Two approaches: *Simulation* or *exact inference*.
  - Given fixed values (data) for some nodes, find the maximum aposteriori (MAP) for some other set of nodes, i.e., the values maximizing their marginal posterior density after fixing the data values. *As an example of this, one might estimate the ML values of parameters in a network specified with unknown parameters*.
- Given data and prior knowlege, one may want to *learn* the structure of a suitable BN or Markov model. Machine learning topic.

Exact posterior inference for graphical models

- Given a model represented as a Bayesian Network (or Markov network), the general goal of inference (as in any Bayesian computation) is to compute the marginal distribution of some variables of interest, conditionally on fixing some other variables, called *data*.
- For a Markov network, fixing some variables produces directly another similar Markov network.
- A Bayesian Network may first be converted to a Markov network.
- A direct way to obtain a marginal distribution in a Markov network is *variable elimination*:
  - Integrating (or summing) out variables in factors.
  - Multiplying together factors.
- Any inference algorithm depends on the basic operations above, but they can be "scheduled" and organized in smart ways, using e.g. "message passing" algorithms.
- Some relevant programs: "Genie" and "Hugin".

Example: The Forward-Backward algorithm

Message passing applied to the following Bayesian Network: A *Hidden Markov Model*



Objective: Compute the marginal posterior distribution of every x_i given data $y_0, \ldots, y_T$: Use $\pi(x_i \mid y_0 \ldots, y_T) \propto \pi(y_{i+1}, \ldots, y_T \mid x_i)\pi(x_i \mid y_0, \ldots, y_i)$ and

1. Forward: For $i = 0, \ldots, T$ compute $\pi(x_i \mid y_0, \ldots, y_i)$ using

$$\begin{aligned} \pi(x_i \mid y_0, \ldots, y_i) &\propto \pi(y_i \mid x_i)\pi(x_i \mid y_0, \ldots, y_{i-1}) \\ &= \pi(y_i \mid x_i) \int \pi(x_i \mid x_{i-1})\pi(x_{i-1} \mid y_0, \ldots, y_{i-1})\, dx_{i-1} \end{aligned}$$

2. Backward: For $i = T-1, \ldots, 0$ compute $\pi(y_{i+1}, \ldots, y_T \mid x_i)$ using

$$\pi(y_{i+1}, \ldots, y_T \mid x_i) = \int \pi(y_{i+2}, \ldots, y_T \mid x_{i+1})\pi(y_{i+1} \mid x_{i+1})\pi(x_{i+1} \mid x_i)\, dx_{i+1}$$