

Financial Risk: Credit risk project

The following assignment should be handed in before 23.59, Sunday the 16-th of December, 2012. Send the report to Alexander.Herbertsson@economics.gu.se. If you write the report in word, then *do not convert the word-file into a pdf-file*

Carefully motivate and explain all terms, expressions and computations. Attach your code in in the report, but also send your computer files (m-files, excel-files, etc) separately, to the above email-adress. If you send in excel-files, make sure to debug them. The most preferable program is matlab, but other programs are also accepted (however, matlab will indeed be the easiest program for implementing this assignment). You are of course encouraged to contact me when you have questions.

Good luck!

Alexander

1. THE MIXED BINOMIAL MODEL INSPIRED BY THE MERTON FRAMEWORK

1.1. Consider a static credit portfolio with 1000 obligors. We model this portfolio as a mixed binomial model inspired by the Merton model with constant default losses. Let the individual default probability for each obligor be $\bar{p} = 4\%$ (for a default within one year, say) and assume that the correlation is $\rho = 10\%$. What is the probability that within one year, there will be more than 100 defaults but less than 200 defaults in our portfolio. Use the large portfolio approximation formula. Repeat the same computation for the nine pairs (\bar{p}, ρ) given in Table 1 and report the result in a table (hint: in matlab, use the built-in functions `normcdf` and `norminv`).

Table 1. The nine pairs (\bar{p}, ρ) . The parameters \bar{p} and ρ are expressed in percent (i.e. $(\bar{p}, \rho) = (3, 25)$ means that $\bar{p} = 0.03$ and $\rho = 0.25$).

$(\bar{p}, \rho) = (4, 15)$	$(\bar{p}, \rho) = (4, 25)$	$(\bar{p}, \rho) = (4, 45)$
$(\bar{p}, \rho) = (8, 15)$	$(\bar{p}, \rho) = (8, 25)$	$(\bar{p}, \rho) = (8, 45)$
$(\bar{p}, \rho) = (16, 15)$	$(\bar{p}, \rho) = (16, 25)$	$(\bar{p}, \rho) = (16, 45)$

1.2. Consider a static credit portfolio with m obligors which we model as mixed binomial model inspired by the Merton framework. The individual one-year default probability is \bar{p} , the individual loss is ℓ , and the default correlation is ρ . Use the large portfolio approximation formula to derive an analytical expression for the one-year $\text{VaR}_\alpha(L)$ as functions of ℓ, m, ρ, \bar{p} and α (see also Equation (32) on slide 25 in the second credit lecture).

1.3. Consider a static credit portfolio with 1000 obligors which we model as mixed binomial model inspired by the Merton framework. Let the individual loss be 60% of the individual notational where each loan have notational 1. Use the formula derived in 1.2 and compute the 1-year $\text{VaR}_\alpha(L)$ and $\text{ES}_\alpha(L)$ for this portfolio where $\alpha = 95\%, 99\%$ and

99.9%. Do this for the nine pairs (\bar{p}, ρ) given in Table 1 and report the result in a table. Hint: compute $\text{ES}_\alpha(L)$ by using e.g. the built-in function `quad` in matlab.

2. COMPARING DIFFERENT METHODS OF COMPUTING CREDIT PORTFOLIO DISTRIBUTIONS IN THE MIXED BINOMIAL MERTON MODEL

2.1. Consider a static credit portfolio with m obligors. We model this portfolio as a mixed binomial model inspired by the Merton model with constant default losses. For three different portfolios $m = 50, 100, 1000$ motivate why the following probabilities

$$\mathbb{P}[N_{50} > 5], \mathbb{P}[N_{100} > 10] \quad \text{and} \quad \mathbb{P}[N_{1000} > 100]$$

are the same when using the large portfolio approximation (LPA) formula. Furthermore, for each pair (\bar{p}, ρ) given in Table 2, compute one of the above probabilities using the LPA formula that is, compute the probability that there will be more than 10% defaults in a portfolio of e.g 50 obligors, and do this for three different scenarios of (\bar{p}, ρ) given in Table 2. Report the result in a table containing the portfolio default probabilities for the three different scenarios of (\bar{p}, ρ) . Hint: Recall that $\mathbb{P}[N_m > k] = 1 - \mathbb{P}[N_m \leq k]$.

Table 2. The nine pairs (\bar{p}, ρ) . The parameters \bar{p} and ρ are expressed in percent (i.e. $(\bar{p}, \rho) = (4, 15)$ means that $\bar{p} = 0.04$ and $\rho = 0.15$).

$(\bar{p}, \rho) = (4, 15)$	$(\bar{p}, \rho) = (8, 25)$	$(\bar{p}, \rho) = (16, 45)$
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2.2. **The normal approximation of the mixed binomial Merton model.** Read the note "Some complementary material to the course "Financial Risk" MVE220/MAS400GU". After this, consider the same problem as in the previous task and recall the central limit theorem approximation for $\mathbb{P}[N_m \leq k]$, i.e.

$$\mathbb{P}[N_m \leq k] \approx \int_{-\infty}^{\infty} N\left(\frac{k + 0.5 - mp(z)}{\sqrt{mp(z)(1-p(z))}}\right) \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz \quad \text{for } k \leq m.$$

where $p(x)$ is given by

$$p(x) = N\left(\frac{N^{-1}(\bar{p}) - \sqrt{\bar{\rho}}x}{\sqrt{1-\bar{\rho}}}\right) \quad \text{for } x \in \mathbb{R}.$$

Now, use the above CLT-formula to compute the following probabilities

$$\mathbb{P}[N_{50} > 5], \mathbb{P}[N_{100} > 10] \quad \text{and} \quad \mathbb{P}[N_{1000} > 100]$$

for three different scenarios of (\bar{p}, ρ) given in Table 2. Report the result as three different tables (one table for each $m = 50, 100, 1000$), each table containing the probabilities for three different scenarios of (\bar{p}, ρ) .

Hint 1: Ignore the fact that the rule of thumb $5 > mp(z)(1-p(z))$ sometimes may be violated.

Hint 2: using the built-in function `quad` in matlab.

Hint 3: Use the integration limits -10 to 10 in `quad`, since the density of a normal cdf can

be treated as zero outside the interval $[-10, 10]$. If you use a larger interval than $[-10, 10]$ quad will complain since $p(z) \approx 1$ for $z > 10$ or $p(z) \approx 0$ for $z < -10$ leading to division by zero (but these complaints can be ignored and matlab will still generate a correct value).

2.3. Computing the exact value using Monte Carlo simulation. Read the note "Some complementary material to the course "Financial Risk" MVE220/MAS400GU". After this, consider the same problem as in the previous task.

Use Monte Carlo (MC) simulations for $n = 10^3, 10^4, 10^5$ to estimate the following probabilities

$$\mathbb{P}[N_{50} > 5], \mathbb{P}[N_{100} > 10] \quad \text{and} \quad \mathbb{P}[N_{1000} > 100]$$

for three different scenarios of (\bar{p}, ρ) given in Table 2. When simulating the Bernoulli random variables X_1, \dots, X_m use the MC-algorithm presented on p.9 in the note "Some complementary material to the course "Financial Risk" MVE220/MAS400GU", i.e. the method with the standard normal random variables Y_1, \dots, Y_m . For each fixed pair m, k , report the MC-simulation result of $\mathbb{P}[N_m > k]$ in a matrix (or table) where each row in the matrix displays the MC-estimation of $\mathbb{P}[N_m > k]$ for a fixed n so that every column shows the estimate $\mathbb{P}[N_m > k]$ for the three different scenarios of (\bar{p}, ρ) . This in order to display how the estimation of $\mathbb{P}[N_m > k]$ improves as n increases, row for row (i.e. the first row contains the MC-estimations for $n = 10^3$, the second row the MC-estimations for $n = 10^4$ etc.). Since there are three pairs $(m, k) = (50, 5), (m, k) = (100, 10), (m, k) = (1000, 100)$, there will be three tables to report, each presented according to the above instructions.

Hint 1: Simulate the probabilities $\mathbb{P}[N_m \leq k]$ and use that $\mathbb{P}[N_m > k] = 1 - \mathbb{P}[N_m \leq k]$. Hint 2: When simulating the random variables Y_1, \dots, Y_m use the built-in matlab-command `randn` in matlab. Use the fact that matlab allows vector-valued comparisons of e.g. the type `x<5` or `x<=5` where `x` is a vector, and where the command `x<5` returns a zero-one vector representing a boolean argument resulting in one (or zero) if the logical statement `x(i)<5` is true (or not true) for entry i in the vector `x`.

2.4. Comparing different methods of computing credit portfolio distributions.

Consider the same problem as in the previous task and use the obtained MC-simulation estimations for $n = 10^5$ as an approximation of *the true value* of $\mathbb{P}[N_m > k]$ for the following cases

$$\mathbb{P}[N_{50} > 5], \mathbb{P}[N_{100} > 10] \quad \text{and} \quad \mathbb{P}[N_{1000} > 100].$$

Compare these MC-simulation estimations for $n = 10^5$ with the corresponding values obtained in Task 2.1 using the LPA method and in Task 2.2 using the CLT method. Report the relative errors for both methods in tables (where the MC-results is thus seen as the true value).

Is one of the methods (i.e. the LPA or CLT method) consistently better than the other? Are some approximations better for some different values of the pair (\bar{p}, ρ) ? What is the role of the portfolio size m ? Carefully discuss your results and your finding as well as your motivations. Do not just display the relative errors without a discussion of your findings and motivations (this type of solutions will not be accepted).