

HOME TO THE LARGEST MULTI-BILLION DOLLAR CORPORATE ACCOUNTING FRAUD IN THE HISTORY OF OUR NATION

CLINTON, MISSISSIPPI -- FORMER CORPORATE WORLD HEADQUARTERS FOR WORLDCOM

my cube was approximately there

Kevin's office was right there

As the WORLDCOM turns.

CORPORATE CRIME, FBI INVESTIGATIONS, MASS LAYOFFS, LOSSES, MERGERS AND MORE.

MY JOB DESCRIPTION:
I started working at WorldCom in September of 1999. While I was there, I was the designer for their Corporate Intranet, which happened to be the busiest intranet site in the world. I also did graphics for the internet site as well. I quit my job soon after Quinn was born in February of 2002.

BITTER: Well, we lost it all! Kevin got laid off after working there for over 8 years!!! We had just bought a house, and had a baby!! Not to mention I had just quit my job (at WorldCom) to be a stay-at-home-mom! We both lost our ENTIRE 401K's and Kevin also lost hundreds of thousands of dollars in vested stock options. Times were tough!

SWEET: This is where we MET!!! This is where we worked when we got married and had our first child! This is where our life together began. So even though we lost all of our money, we gained ALL of those things that money just CANNOT BUY!! Like true love, a best friend, and family! I wouldn't trade this WorldCom experience for anything!!!!

KEVIN'S JOB DESCRIPTION:
Kevin started working at WorldCom when it was actually still LDDS. It's hard to just sum up ALL of what he did for them. He was there through over 60 mergers and acquisitions and played a major part in integrating the systems of all of the new companies. He was also over all of their Corporate Internet and Intranet Systems. Kevin was laid off from WorldCom in April of 2002, which was 2 months after I quit.

WORLDCOM
KIMBERLY SMITH
10/21/2002

WORLDCOM
Corporate
KEVIN CROTHERS

Financial Risk
2-rd quarter 2012/13
Tuesdays 10.15 – 12.00 and
Thursday 13.15 – 15.00 in MVF31 and Pascal, resp.

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“As an alternative to the traditional 30-year mortgage, we also offer an interest-only mortgage, balloon mortgage, reverse mortgage, upside down mortgage, inside out mortgage, loop-de-loop mortgage, and the spinning double axel mortgage with a triple lutz.”



Gudrun January 2005
326 MEuro loss
72 % due to forest losses
4 times larger than second largest

This course:

- Learn some fun things about financial risks
- Learn some basic risk management tools from Extreme Value Statistics
- See some basic quantitative Credit Risk models

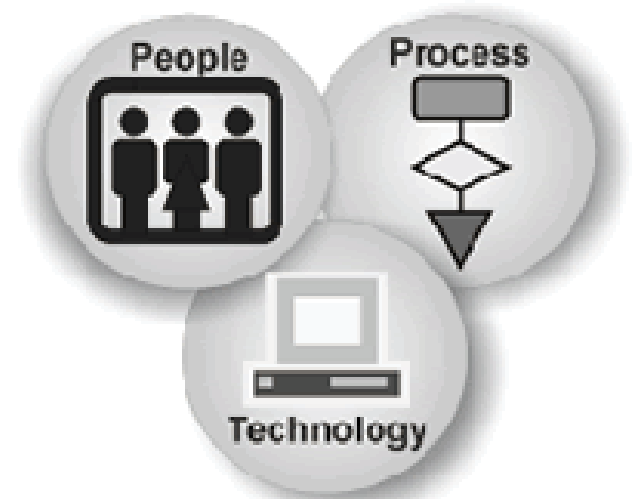
But not

- A complete systematic account of financial risk management
- Financial time series modeling
- Black-Scholes option pricing methods
- Macroeconomics

and

There are risks which cannot be handled by mathematical models!!

- Credit risk
- Market risk
- Operational risk
- Insurance risk
- Liquidity risk
- Reputational risk
- Legal risk
- and so on ...



Operational risk

Risk factors

$L = \text{"-P/L"} = \text{Loss} - \text{Profit} = f(X_1, \dots, X_d)$, X_1, \dots, X_d risk factors, *e.g.* exchange rates, interest rates, index movements, stock prizes,

Example: Linear portfolio, α_i shares of stock i , stock prize $S_{t,i}$

$$L = - \sum_{i=1}^d \alpha_i S_{t+1,i} + \sum_{i=1}^d \alpha_i S_{t,i} = - \sum_{i=1}^d \alpha_i S_{t,i} \left(\frac{S_{t+1,i} - S_{t,i}}{S_{t,i}} \right)$$

How big is the risk?

Quantitative risk management methods:

- Historical data or historical simulation
- Stress testing (“scenarios”)
- Sensitivity measures (“the greeks”)

- Full statistical modeling (often multivariate normal + linear portfolio)
- Semiparametric modeling of the “tails” of the *loss-profit* distribution (univariate Extreme Value Statistics) (*this course*)
- Semiparametric modeling of the tails of the multivariate distribution of the risk factors (multivariate Extreme Value Statistics, “Copulas”) + computation of the *loss-profit* distribution analytically or via stochastic simulation

How **big** is the risk??

Don't look at the stars with a microscope --- and don't use statistical methods tailored to means and typical behavior to study extreme occurrences:

Use Extreme Value Statistics! (if not -- you will not see the important things)

Normal statistics: based on the normal distribution, concerned with averages and typical behavior

Extreme Value Statistics: based on Extreme Value and Generalized Pareto Distributions, concerned with rare events

Model checking and adjustment

- **Stationarity**
- Independence
- Distributional fit
- Period and threshold choice

Statistical model



Data analysis

- Choice of distributions
- Estimation (often of quantiles)
- Confidence intervals
- “Prediction intervals”
- Hypothesis testing
- Regression and dependence modeling (*not this course*)
- Prediction (*not this course*)
- ***Understanding!!***

refresh your basic statistics knowledge!

Basic EVS:

--- Block Maxima: EV (= GEV) distribution for maxima

--- Peaks over Thresholds: GP distribution for tails

Why?

- **stability:** maxima of variables which are EV distributed are also EV; going to higher levels preserves the GP distribution of exceedances (cf. “standard statistics: sums of normally distributed variables have a normal distribution”)
- **asymptotics:** maxima of many independent variables are often (approximately) EV distributed; asymptotically tails are GP when maxima are EV (cf. “standard statistics: sums of many small “errors” are often (approximately) normally distributed the “central limit theorem”)
- **”transition”:** easy to go back and forth between GP and EV

but don't believe in models blindly

The Block Maxima method *(Coles p. 45-53)*

the Extreme Value (EV) distribution:

$$G(x) = \exp\left\{-\left(1 + \gamma \frac{x-\mu}{\sigma}\right)_+^{-1/\gamma}\right\}$$

the special case $\gamma = 0$ is the Gumbel distribution

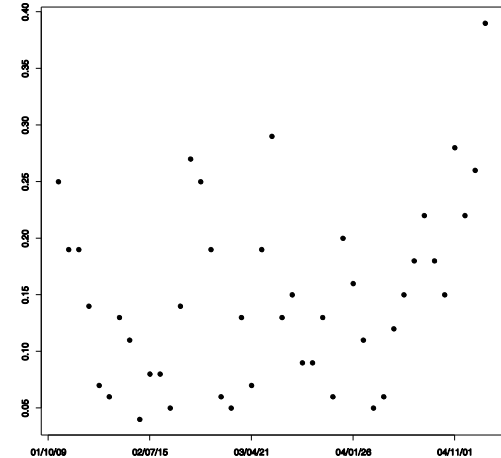
$$G_0(x) = \exp\left\{-\exp\left\{-\frac{x-\mu}{\sigma}\right\}\right\}$$

Fit to the measured block (=weekly, or monthly, or yearly, or ...) maxima, assuming independence, using maximum likelihood. Find p -th quantile from top (= value such that the risk that it is exceeded is $100p$ %) by solving

$$\hat{G}(x_p) := 1 - \hat{G}(x_p) = p,$$

where $\hat{}$ means that parameters are replaced by their estimated values.

confidence intervals via profile likelihood or delta method *(later lectures)*



Maximum long term US interest rate, constant maturity, nominal 1 month, percentage points

Some mathematics behind the Block Maxima Method:

X_1, X_2, \dots independent identically distributed random (i.i.d.) variables with distribution function (d.f.) F (e.g. daily interest rates, c.f. previous slide)

$M_n = \max\{X_1, \dots, X_n\}$ = the maximum of the first n variables (e.g. $n = 30$ and X daily interest rates gives plot on previous slide)

$$\begin{aligned} P(M_n \leq x) &= P(X_1 \leq x, \dots, X_n \leq x) \\ &= P(X_1 \leq x) \times \dots \times P(X_n \leq x) = F(x)^n \end{aligned}$$

Exercise: Show that the EV distributions are *max-stable*, i.e. that the maximum of n i.i.d. EV-distributed variables also have an EV distribution, i.e. that if $G(x) = \exp\left\{-\left(1 + \gamma \frac{x-\mu}{\sigma}\right)_+^{-1/\gamma}\right\}$, then

$$G(x)^n = \exp\left\{-\left(1 + \gamma \frac{(x-\mu_n)/\sigma_n - \mu}{\sigma}\right)^{-1/\gamma}\right\} = G\left(\frac{x-\mu_n}{\sigma_n}\right)$$

and find μ_n, σ_n .

Theorem: *The distribution function G is max-stable if and only if it is an EV distribution*

In the previous exercise it was shown that the EV distributions are max-stable, which proves half of this theorem. The other half consists of solving the functional equations

$$G(x)^n = G\left(\frac{x - \mu_n}{\sigma_n}\right), \text{ for } n = 1, 2, \dots$$

to find that the EV distributions are the only solutions.

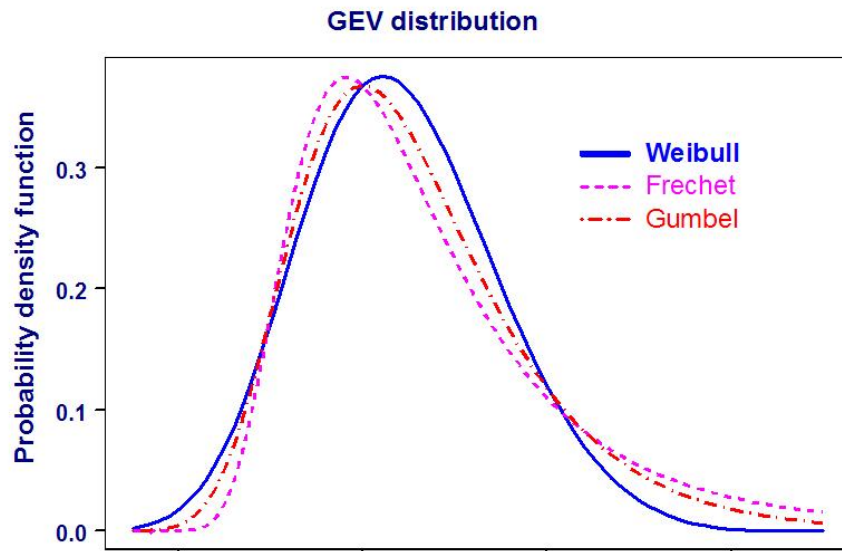
Theorem: *If there are constants $b_n > 0, a_n$ such that*

$$P\left(\frac{M_n - a_n}{b_n} \leq x\right) \rightarrow G(x), \text{ as } n \rightarrow \infty \text{ for all } x$$

then $G(x)$ is an EV distribution. (cf. the central limit theorem)

This is proved by showing that it follows that $G(x)$ must be max-stable

Challenge: Prove this!



$$g(x) = \frac{d}{dx}G(x) = \frac{1}{\sigma} \left(1 + \gamma \frac{x-\mu}{\sigma}\right)_+^{-1/\gamma-1} \exp\left\{-\left(1 + \gamma \frac{x-\mu}{\sigma}\right)_+^{-1/\gamma}\right\}$$

Densities of the generalized Extreme Value distribution

$\gamma > 0$ Frechet distribution, finite left endpoint of distribution: $x > \mu + \frac{\sigma}{\gamma}$

$\gamma = 0$ Gumbel distribution, unbounded support

$\gamma < 0$ Weibull distribution, finite right endpoint of distribution: $x < \mu + \frac{\sigma}{|\gamma|}$

the distribution is “heavytailed” for $\gamma > 0$: then moments of order greater than $1/\gamma$ are infinite/don’t exist; thus if $\gamma > 1/2$ then the variance doesn’t exist, if $\gamma > 1$ then the mean doesn’t exist either

Exercise: For each of the following three distributions, show that, with the given norming constants a_n, b_n , there is a d.f. $G(x)$ such that

$$P\left(\frac{M_n - a_n}{b_n} \leq x\right) \rightarrow G(x), \text{ as } n \rightarrow \infty \text{ for all } x,$$

and find this $G(x)$.

- 1.) The exponential distribution $F(x) = 1 - e^{-\lambda x}$, $\lambda > 0$, $x \geq 0$
(use $a_n = \frac{\log n}{\lambda}$, $b_n = 1/\lambda$)
- 2.) The Pareto distribution $F(x) = 1 - \left(\frac{\kappa}{\kappa+x}\right)^\alpha$, $\kappa, \alpha > 0$, $x \geq 0$
(use $a_n = \kappa n^{1/\alpha} - \kappa$, $b_n = \kappa n^{1/\alpha} / \alpha$)
- 3.) The uniform distribution $F(x) = x$, $0 \leq x \leq 1$
(use $a_n = 1 - 1/n$, $b_n = 1/n$)

(A perhaps unnecessary explanation) **What does**

$$P\left(\frac{M_n - a_n}{b_n} \leq x\right) \rightarrow G(x), \text{ as } n \rightarrow \infty \text{ for all } x,$$

mean in practice? That $P\left(\frac{M_n - a_n}{b_n} \leq x\right) \approx G(x)$, for large n , or,

with $y = b_n x + a_n$ and $G(x) = \exp\left\{-\left(1 + \gamma \frac{x - \mu'}{\sigma'}\right)^{-1/\gamma}\right\}$, that

$$\begin{aligned} P(M_n \leq y) &\approx G\left(\frac{y - a_n}{b_n}\right) = \exp\left\{-\left(1 + \gamma \frac{y - (a_n + b_n \mu')}{b_n \sigma'}\right)^{-1/\gamma}\right\} \\ &= \exp\left\{-\left(1 + \gamma \frac{y - \mu}{\sigma}\right)^{-1/\gamma}\right\}, \text{ for } \mu = a_n + b_n \mu', \sigma = b_n \sigma'. \end{aligned}$$

Since all the parameters are unknown anyway, we are left with the problem of estimating μ, σ from data, *i.e.* with the Block Maxima method.