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The Capital asset pricing model and the Arbitrage pricing theory

Authors:
Trang Nguyen
Olivia Stalin
Ababacar Diagne
Leonard Aukea

Supervisors:
Pr. Holger Rootzen
Dr. Alexander Herbertsson

This report has been written and analyzed by all the group members jointly.

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The Capital Asset Pricing Model and the Arbitrage Pricing Theory

Leonard Aukea, Ababacar Diagne, Trang Nguyen, Olivia Stalin

Abstract

In this work we review the basic ideas of the Capital Asset Pricing Model and the Arbitrage Pricing Theory. Furthermore, we exhibit the practical relevance and assumptions of these models. We show what make them successful for the pricing of assets. Indeed, the drawback and limitations of these models will be addressed as well.

Keywords: Capital Asset Pricing Model, Arbitrage Pricing Theory, asset pricing.

1 Introduction

Based on the pioneering work of Markowitz (1952) and Tobin (1958) for risky assets in a portfolio, Sharpe (1964), Lintner (1965) and Mossin (1966) derived a general equilibrium model for the pricing of assets under uncertainty, called the Capital Asset Pricing Model (CAPM).

CAPM is a well-known and accepted single factor model, after four decades CAPM is still one of the main alternatives in the estimation of expected return or cost of equity for individual stocks (commodity derivatives, energy/electricity markets, etc.) and other financial securities. This task is central to many financial decisions such as those relating to portfolio optimization, capital budgeting, and performance evaluation. The measure of risk in the CAPM is given by the security's covariance with the market portfolio, the so-called market beta.

Rather, the CAPM quantifies the expected rates of return of an asset with its relative level of market systematic risk (beta). This explains why the

CAPM is called often a single factor model. Another model for the estimation of asset returns is the Arbitrage Pricing Theory (APT). To improve the discrepancy of the CAPM, the APT model was proposed by Stephen Ross (1976) as a general theory of asset pricing. His theory predicts a relationships between the returns of a single asset as a linear function of many independent macroeconomic factors. In a historical context, the CAPM was the first coherent framework answering the question of how expected returns and risk were related.

In fact, as noted by authors in [15], the attraction of these model is that it offers powerful and intuitively pleasing predictions about how to measure risk and the relation between expected return and risk. The CAPM and APT are simple asset pricing tools comparing to other probabilistic and stochastic models.

Elsewhere, following authors in [12], the APT has generated an increased interest in the application of linear factor models in the study of capital asset pricing and a large academic literature [15, 13, 14, 10]. As an illustration some extensions and some modified versions of the CAPM have been developed. For instance, the Conditional CAPM (CCAPM) models the time-varying property of the distribution of stock returns (cf [18]). In addition, an application of the consumption-based CAPM (pricing performance in seven industry sub-sectors in the Taiwan stock market) is presented in [11]. Here, the risk of a security is measured by the covariance of its return with per capita consumption and is called consumption beta. Theoretically, the consumption beta offers a better measure of systematic risk.

To name only a few, using stock listed in the Korean stock exchange, authors in [21] present a comparative study by considering different versions of CAPM and the APT models such as: CAPM, APT-motivated models, the Consumption-based CAPM, Intertemporal CAPM-motivated models, and the Jagannathan and Wang conditional CAPM model.

Besides, the CAPM and the APT have provided interesting and challenging research topics [17, 8, 5, 27, 26, 7]. For instance, Bartholdy and Peare in [5] conducted a comparative study of the performance of this two models for individual stocks. They make use of the Fama and French three-factor model for the APT. As a result, they show that the Fama French model is at best able to explain, on average, 5% of differences in returns on individual stocks despite the favor of the CAPM by practioners. Moreover, when the risk levels are given by coherent risk measures such as VaR, CVaR, and WCVaR, authors in [4] presented the optimal portfolio choice problems and

some extensions of the classic Arbitrage Pricing Theory (APT) and CAPM. We refer the interested reader about these risk measures to [25, 16]. In [2], the so called conditional CAPM is discussed. Likewise, by making use of a Kalman filter, Tobias and Frazoni model the conditional beta and their approach circumvents recent criticisms of this risk measure. They tackle a number of the issues by assuming that betas change over time following a mean-reverting process.
Moreover, Hwang and collaborators in [20] present a sharp idea of using the credit spread as an option-risk factor and explain some limitations of the traditional CAPM. They contribute to the option-risk CAPM literature by using bond-credit-spread data as a proxy for default risk to control for the option-risk characteristic of stocks. Their option-risk version of CAPM resembles the conditional CAPM. Another application of the CAPM and APT on the private equity asset class can be found in the recent paper [9]. Since these asset by nature are illiquid, the traded liquidity factor is included in the APT model. Even tough, an outstanding review and historical walk-through of the CAPM is presented in [22]. The author claims that the CAPM deviations is not due just to missing risk factors, hence the APT cannot be an attempt to correct it.

Very recently, economically meaningful results with important policy implications are found by By Aabo and co-authors in [1]. They introduce the degree of the internationalization as a new corporate risk and illustrate their results considered Scandinavian multinational firms. Hence, one can make use of this factor in the APT.
Inter alia, Östermark in [3] revisited the portfolio efficiency of the APT and the CAPM in Two Scandinavian Stock Exchanges (Finnish and Swedish). As a finding, he demonstrates that the multifactor APT is more powerful in predicting Finnish than Swedish stock returns, whereas the contrary holds for the single factor CAPM.

Among others, the main aim of this present work is to present the basic idea of the CAPM and the APT and we demonstrate the practical relevance and assumption of these models show what make them successful for the pricing of assets. Indeed, the drawback and limitations of these models will be addressed as well.

The structure of this work is organized as follows. After the introduction in first Section, we set up some preliminaries in Section 2. Section 3 is devoted presentation of the CAPM and its derivation. The APT is discussed in Section 4 before the conclusion and discussions in Section 5.

2 Preliminaries

To ease the understanding in the sequel we provide some concepts in finance and definitions that will be of importance.

2.1 Markowitz portfolio theory

In the framework Markowitz mean-variance portfolio theory, the rate of returns of a stock is considered as random variables. Assume, we would like to build a compulsive portfolio from n given assets $A_i((r_i), \sigma_i^2)$ where $E(r_i)$ is the expected return and σ_i^2 the variance of the return. Here the variance of the rate of return of an instrument is taken as a surrogate for its risk (volatility). On each asset A_i, we invest a proportion w_i such that $\sum_{i=1}^{n} w_i = 1$. The selection of an investment opportunity in the Markowitz theory is an optimization problem. It consists of finding the optimal set of weights w_i such that the portfolio achieves an acceptable baseline expected rate of return under a minimal volatility. Without loss of generality, in the Markowitz' basic principle investors will keep a risky security only if the expected return is sufficiently high enough to compensate them for assuming the risk. This is know as 'Risk and Return trade-off'. We set the vector of investment proportions $w = (w_1, w_2, \cdots, w_n)$ and we denote the covariance matrix between the assets by $\mathbf{C}$. The expected return of our portfolio is given by

$$E(r_P) = \sum_{i=1}^{n} w_i E(r_i) \tag{1}$$

and its variance

$$\sigma^2(r_P) = w^T \mathbf{C} w \tag{2}$$

An optimal portfolio will be any investment strategies solving the problem

$$\begin{cases} \min \dfrac{1}{2} w^T \mathbf{C} w \\ \text{subject to } E(r_P) \geq r_b \\ \displaystyle\sum_{i=1}^{n} w_i = 1. \end{cases}$$

2.2 Efficient Frontier

In a risk-return framework, a Markowitz Efficient Frontier, is the curve that shows all the best combinations of securities that yield the maximum expected return for a given level of risk. It is defined also as the set of portfolios that minimizes the risk subject to a given expected return. We are looking for the investment policy that solves the previous optimization problem. Since investors are generally risk averse, they will always invest in an efficient portfolio. The efficient frontier (Pareto front?) traces out a increasing curve in the risk-return plane as depicted here after in Figure 1.

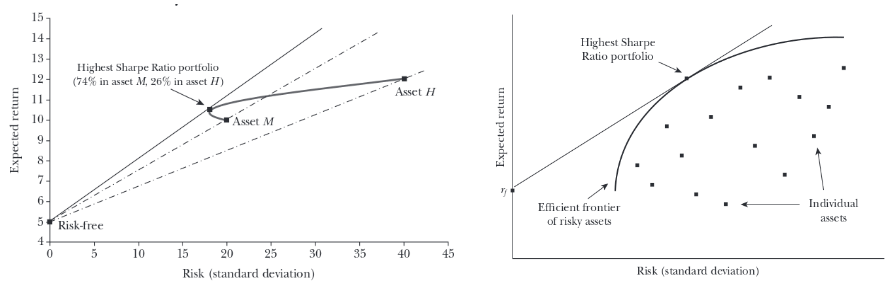

Figure 1: The efficient frontier (EF) with two risky assets (left) or many risky assets (right). The Capital market line (CML)tangent to the EF curve gives the optimal portfolio (source [23]).

The portfolio on the EF that has the highest Sharpe Ratio, which will be the point where the CML is just tangent to the EF will determine the optimal portfolio.

2.3 Arbitrage

An arbitrage portfolio is a portfolio whose value $V(t)$ satisfies the following properties, for some time horizon $T > 0$:

- $V(0) = 0$ almost surely,
- $V(T) \geq 0$ almost surely,
- $\mathbb{P}(V(T) > 0) > 0$ where $\mathbb{P}$ is a given probability measure.

In other words, a portfolio presents an arbitrage opportunity if it does not require any initial wealth to hold it and guarantees a strictly positive return at time T. Hence an arbitrage portfolio or simply arbitrage can be considered as a risk-free investment strategy. Basically an arbitrage represents a deterministic money making instrument (which entails no risk).

We introduce this fundamental concept of arbitrage to ease the understanding in the sequel. In the playground of mathematics and finance, there is a large number of work devoted to this concept of arbitrage. Nevertheless, in sight of the complexity of modern markets arbitrage opportunities may exist. They are characterized by practitioners as a result of market inefficiencies. The life time of an arbitrage is very short in time since they are quickly exploited and traded away by investors, which actually stabilizes the market. Without loss of generality, Björk in [6] interprets the existence of arbitrage portfolio as a serious case of mispricing on the market. Let's mention that this mispricing is relative to a given asset pricing model.
With these pricing models included the CAPM, a common theoretical assumption is the arbitrage-free principle which can be interpreted as: asset prices in a financial market are such that no arbitrage opportunities can be found. Its worthy to note that the arbitrage-free principle plays a key role in finance and stand as the foundation of option pricing theory.

3 The Capital Asset Pricing model

As already stated in the introduction, the CAPM aims at a practical approach to stock valuation. It is a model that gives you an appropriate expected rate of return for each asset given the relevant risk characteristic of that asset versus the market called beta. This latter factor beta measures the quantity risk that cannot be diversified away. Like any other model, the CAPM is constructed from a set of underlying assumptions about the real world. The fundamental assumptions of the CAPM are as follows:

3.1 Assumptions of CAPM

In what follows, we review the assumptions behind the CAPM framework.

1. The financial markets are competitive
2. All investors plan to invest over the same time horizon
3. There is no distortionary taxes or transaction costs (markets are frictionless)

4. The investments are limited to publicly traded assets with unlimited borrowing and lending at the risk-free rate and the market portfolio consists of all publicly traded assets.

5. All investors like overall portfolio reward (expected return) and dislike overall portfolio risk (variance or standard deviation of return)

6. Everyone either has quadratic utility or has homogeneous beliefs concerning the distribution of security returns.

From these assumptions, one clearly sees that the CAPM is built under a perfect competition assumption of microeconomics. The price of assets are unaffected by the trades of investors which hold a small wealth compared to the total endowment of all investors. We further observe that the total return of any investor's portfolio is a summation from two components: the risk free assets and the risky market assets. This is due to the possibility of lending and borrowing at the free rate. Furthermore, all the information is available at the same time to all investors.
Besides its practical use, we note that the CAPM has many unrealistic assumptions. For instance, the perfect competition assumption of microeconomics does not hold, the forces of supply and demand determine the prices of asset in reality between buyers and sellers. Since investments are limited to a universe of publicly traded financial assets, this assumption rules out many types of investments. Moreover, we know that investors are in different tax brackets and that this may govern the type of assets in which they invest. In other terms, naturally, there is no homogeneous expectations or beliefs between investors. But this assumptions is crucial in the CAPM since if the investors do not have similar expectations there will be no homogeneity in their conception. Another highly unrealistic assumption is the fact that investors should have identical time horizons, which obviously is not the case. This assumption is a consequence of the CAPM being a single period model. As an alternative, continuous time models are used to get over the above difficulty of single periods. In summary, we may say that the these assumptions represent a very simplified and idealized world, nevertheless they are crucial to arrive at the original and basic form of the CAPM as stated in the following theorem.

Theorem 1 *We denote by M the market portfolio with expected return $E(r_M)$ and variance σ_M^2. We assume there exist a risk-free return r_f.*

For any asset i,

$$E(r_i) = r_f + \beta_i(E(r_M) - r_f)$$

where

$$\beta_i = \frac{Cov(r_i, r_M)}{\sigma_M^2}$$

is called the beta of the asset i. The quantity σ_M^2 is the variance of the market portfolio.
More generally, for any portfolio $p = (\alpha_1, ..., \alpha_n)$ of risky assets, its beta can be computed as a weighted average of individual asset betas,

$$E(r_p) = r_f + \beta_p(E(r_M) - r_f)$$

where

$$\beta_p = \sum_{i=1}^{n} \alpha_i \beta_i.$$

The beta value serves as an important measure of risk for individual assets (portfolios) that is different from σ_i^2. It measures the non-diversifiable part of risk called systematic risk. Besides, the CAPM can be seen as a single factor model and the factor β can be appreciated as the factor sensitivity of the asset's return to the return of the market portfolio. To name only a few, one aspect of the CAPM is that the expected return of an asset does not depend on its stand-alone risk but depends on its sensitivity towards the market. Another worth noting aspect of the CAPM is its role in testing whether stock markets are efficient in terms of expected return.

3.2 Derivation of the CAPM

This subsection is devoted to the derivation of the CAPM. There are several methods to arrive at the CAPM. Here, we follow ideas presented in [19] that use an algebraic approach based on the Markowitz (financial engineering mean-variance portfolio theory from Markowitz) portfolio theory. We consider the market portfolio (M) and a risky asset (A_i). Investing a fraction y of my investment funds in the the risky asset (A_i) and the fraction $1 - y$ in the market portfolio, the expected return and variance of my compulsive portfolio ($P = (M, A_i)$) writes as:

$$E(r_p) = yE(r_i) + (1 - y)E(r_M) \tag{3}$$

$$\sigma_p^2 = y^2\sigma_i^2 + (1-y)^2\sigma_M^2 + 2y(1-y)\sigma_{iM} \tag{4}$$

These quantities are a function of the investment proportion in the risky asset y. Taking the derivative of $E(r_p)$ and σ_p with respect to y, we obtain:

$$\frac{\partial E(r_p)}{\partial y} = E(r_i) - E(r_m), \tag{5}$$

$$\frac{\partial \sigma_p}{\partial y} = \frac{2y\sigma_i^2 - 2(1-y)\sigma_M^2 + 2(1-2y)\sigma_{iM}}{2\sqrt{y^2\sigma_i^2 + (1-y)^2\sigma_M^2 + 2y(1-y)\sigma_{iM}}}. \tag{6}$$

Since all investors use identical analysis of the same universe of assets, in general equilibrium the market portfolio (M) already includes the risky asset (A_i) (according to its market value weights). Then the proportion y can be interpreting as measuring excess demand for the risky asset. Under general equilibrium, no general equilibrium will exist i.e we set y to zero. Hence the partial derivatives rewrites as:

$$\left.\frac{\partial E(r_p)}{\partial y}\right|_{y=0} = E(r_i) - E(r_m), \tag{7}$$

$$\left.\frac{\partial \sigma_p}{\partial y}\right|_{y=0} = \frac{\sigma_{iM} - \sigma_M^2}{\sigma_M}. \tag{8}$$

The market price of risk, the equilibrium risk-return trade-off will therefore be the ratio

$$\frac{\frac{\partial E(r_p)}{\partial y}}{\frac{\partial \sigma_p}{\partial y}} = \frac{E(r_i) - E(r_M)}{\sigma_{iM} - \sigma_M^2}\sigma_M. \tag{9}$$

Consider an investment scenario in the risk-free asset (F) and the market portfolio (M). Due to the capital market line (CML), we have a relation for the expected return of the portfolio:

$$E(r_p) = r_f + \frac{E(r_M) - r_f}{\sigma_M}\sigma_p. \tag{10}$$

Here the equilibrium, the market price of a risk (risk-return trade-off) is given by:

$$\frac{E(r_M) - r_f}{\sigma_M}. \tag{11}$$

Seen that in equilibrium, the marginal price-of-risk for all assets must be equal (for none arbitrage reason), we get by equating equations (9) and (11) the following relation

$$\frac{E(r_M) - r_f}{\sigma_M} = \frac{E(r_i) - E(r_M)}{\sigma_{iM} - \sigma_M^2}\sigma_M. \tag{12}$$

From this latter relation, one obtain:

$$E(r_i) = r_f + \frac{\sigma_{iM}}{\sigma_M^2}(E(r_M) - r_f). \tag{13}$$

We define the coefficient $\beta_i = \dfrac{\sigma_{iM}}{\sigma_M^2}$ and rewrite (13) as

$$E(r_i) = r_f + (E(r_M) - r_f)\beta_i. \tag{14}$$

This linear relationship between risk and return known as security market line (SML) is the traditional CAPM model derived by Sharpe in 1964. Figure 2 shows a plot of the SML. The beta of the asset (A_i), β_i, measures the quantity of risk exposure in asset (A_i) while $(E(r_M) - r_f)$ is the market price of risk and $\dfrac{\sigma_{iM}}{\sigma_M^2}(E(r_M) - r_f)$ determines the market risk premium.

One can view β_i as a measure of non-diversifiable risk, the part of risk correlated with the market that one can not reduce by diversifying. This measure is also called the market risk or systematic risk. Often, the stocks in big name companies which are deeply related to the general market are expected to have high beta values.

As explained in [24], it makes sense that the CAPM only rewards investors for their portfolio's responsiveness to the market. The risk that can be diversified away is often called idiosyncratic or specific risk. The part that cannot be escaped is often referred to as systematic risk or market risk. Beta is a measure of this risk. Thus, even efficient portfolios will be exposed to covariance risk.

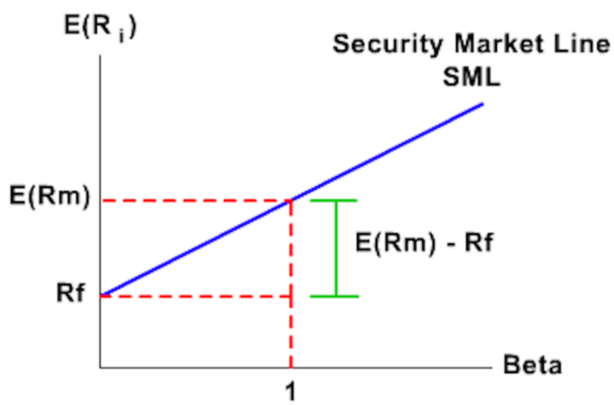

Figure 2: The security market line. We see clearly for the market portfolio we have $\beta_M = 1$.

For the financial analyst, constructing the market portfolio would be a difficult and unrealistic task in sight of the number of stocks in a real financial market. To overcome this, index funds (index tracker or mutual fund) have been created as an approximation of the market portfolio. This index is made up by the most traded (dominant) assets in the market. The assets capture the essence of the given market M, in other terms it mirrors the market performance. The most commonly known index funds, the S&P 500 Index Fund (Standard Poor's 500-stock index). In the Stockholm Stock Exchange, the OMX Stockholm 30 ($OMXS30$) is the stock market index It is a capitalization-weighted index that consists of the 30 most-traded stock classes (see also Nikkei225, DAX30, CAC40, etc).

3.3 Pricing assets

Let the price of an asset at $t = 0$ be $P = X_0$. We set the payoff at $Q = X_1$ at $t = 1$, expected payoff to $E(X_1) = \bar{Q}$ at $t = 1$. We denote the expected return $\bar{r} = E(r_i)$. By definition we have

$$\bar{r} = \frac{E(X_i) - X_0}{X_0} = \frac{\bar{Q} - P}{P}$$

When using $\bar{r}$ as the discount rate, P can be expressed as,

$$P = \frac{\bar{Q}}{\bar{r} + 1}.$$

From the CAPM formula we have

$$E(r_i) = r_f + \beta(\bar{r}_M - r_f)$$

Hence we obtain:

$$P = \frac{Q}{1 + r_f + \beta(\bar{r}_M - r_f)}$$

When using $r = \dfrac{Q - P}{P} = \dfrac{Q}{P} - 1$ then

$$Cov(r, r_M) = Cov((Q/P) - 1, r_M) = Cov((Q/P), r_M) = \frac{1}{P} Cov(Q, r_M) = \sigma_M^2 \beta_i.$$

And since $\beta = Cov(r, r_M)$, we obtain,

$$\beta = \frac{1}{P}\Big(\frac{Cov(Q, r_M)}{\sigma_M^2}\Big).$$

And inserting this in the CAPM formula gives us,

$$P = \frac{\bar{Q}}{(1 + r_f + \frac{1}{P}(Cov(Q, r_M)/\sigma_M^2)(\bar{r}_M - r_f)}.$$

P expresses the price of the present value of an expected pay off.

3.4 Limitations of the CAPM

The CAPM model offers a linear relationship between the systematic risk and expected return for assets. The availability of the inputs and its original simplicity make the CAPM an attractive and well-accepted tool for estimating the expected return of securities. The fact that the model considers the systematic risk β is one of its great aspects. Namely, systematic risk is an important factor since it often cannot be completely alleviated. Notwithstanding, the CAPM makes some non trivial postulates that drive to many drawbacks in the real world.

Inter alia, the CAPM assumes the existence of a perfect financial market, where there are no restrictions on investments in terms of income taxes, transaction costs etc. Obviously, this is far from the truth. This lack of a perfect market may induce an additional risk to the investors when they bump into market regulations. The CAPM model assumes unlimited borrowing and lending of the risk free ratio r_f, and also that r_f has same rate for all investors. In reality individual investors are not allowed to borrow and lend with the same rate as the government. The postula may lead to serious issues in the valuation. Beyond the difficulties to estimate β another issue of the CAPM is related to the return of the market. In fact, a problem arises when the market return at a given time has a negative value. Furthermore, the return of the market is not a proper representation of a future market return. To correct these drawbacks, a large amount of research have been conducted and several necessary improvements of the CAPM have been proposed in the literature. These adjustments yield to multi-factor model that consider not just beta but many source of risk related to the asset. Among other, the so-called arbitrage pricing theory (APT) is have been a very attractive alternative of the CAPM.

In constrast to the CAPM, the APT uses a finite number of factors and the expected return of an asset will be related to its exposure to each of these factors. In addition, the APT has more flexible assumption requirements than the CAPM. In the following section present a review of the APT.

4 The Arbitrage Pricing Theory

The CAPM, equilibrium model of asset pricing, asserts that securities have different expected returns just because they carry distinct betas. Per contra, there exists an alternative model of asset pricing that was developed by Stephen Ross and has more flexible assumption requirements. This model is known as Arbitrage Pricing Theory (APT). One primary assumption of the APT entails the use of arbitrage portfolio. This latter can be seen as an investment tactic that evolves a short position on a security at a high price and a simultaneously long position of the same security or its equivalent at a low price. In effect, whenever they are discovered, investors have an incentive to take advantage of arbitrage portfolio and this is an essential logic in the APT.

As a result, the APT is a more generalized version of CAPM that allows the modeler to extend the CAPM by adding additional macroeconomic factors to the model. The APT can be qualified as an "open-source" model. Namely, the reason is that the APT does not specify either the number or identity of the factors that drive the expected returns of an asset.

Focused on the interest rate, inflation and other aspects of the economy such as GDP growth, oil prices etc, a lot of research has been conducted in order to identify potential factors. There exists several versions of the APT in the literature. For instance we cite the Fama and French Three Factor-Model [13] and in 2011 Chen, Novy-Marx and Zhang published an alternative three-factor model [10], their model fixes some anomalies of the Fama French. More recently (2015), Fama and French proposed in [14] a five-factor model directed at capturing the size, value, profitability, and investment patterns in average stock returns performs better than the three-factor model of Fama and French [13]. These models present good performances and are equipped with sharp economic intuitions.

4.1 Assumptions of the APT

At the difference of the CAPM, the APT does not assume that investors hold efficient portfolios. However it holds three underlying assumptions which are,

1. Asset returns are explained by systematic factors.
2. Investors can build a portfolio of assets where specific risk is eliminated through diversification.
3. No arbitrage opportunity exists among well-diversified portfolios. If any arbitrage opportunity exists, they will be exploited away by investors.

The CAPM is not equipped to determine the current price of stocks. Its only efficiency is to return a stock's expected return.

4.2 The APT equation(s)

Unlike in the CAPM, the APT assumes that the return of an asset is generated by a multiple factors model. Each factor can be viewed as a specific beta coefficient towards a specific risk premium. Several researchers have investigated stock returns and have requested the use of three to five factors in the APT. For instance Stephen Ross and collaborators have identified the following factors:

- rate of inflation
- growth rate in industrial production

- spread between long-term and short-term interest rate
- spread between high-grade and low-grade bond

Other authors recommend to use the rate of interest, rate of change in oil price and even rate of growth in defense spending. Furthermore, the market index is often used as a factor in some versions.
Therefore in the case of n factors, the APT equation writes as

$$E(r_i) = \lambda_0 + \beta_1^i \lambda_1 + \beta_2^i \lambda_2 + \cdots + \beta_n^i \lambda_n \tag{15}$$

in the n-factors model, λ_j are the factors and β_j^i the corresponding sensitivities. For the purpose of calibration λ_0 is usually take as the risk free rate because it has no sensitivity. Equation (15) in many framework, is written under the form:

$$E(r_i) = r_f + \beta_1^i(\delta_1 - r_f) + \beta_2^i(\delta_2 - r_f) + \cdots + \beta_n^i(\delta_n - r_f). \tag{16}$$

Here r_f denotes the risk free rate and we have n risk premiums with their sensitivities (specific "betas" of the stock i). Some studies show that these factors move with the market portfolio. As already stated above, we presented some empirical models for the APT. For instance, the Fama-French three factors model writes as:

$$E(r_i) = r_f + \beta_m^i E(r_M - r_f) + \beta_{SMB}^i E(SMB) + \beta_{HML}^i E(HML), \tag{17}$$

where

- $E(r_M - r_f)$ is the excess expected return of the market
- $E(SMB)$ and $E(HML)$ stand respectively as the expected return of size factor and the BE/ME factor.

Lets mention that in their design, they work with two variables that are represented by two portfolios named small minus big (SMB) and high minus low (HML). They also consider a book-to-market equity (BE/ME) factor. For more details, we refer the reader to [13]. The three different "betas" can be estimated by running time series regressions on historical data. Moreover, Chen and co-authors considered two additional factors on the CAPM. The new factors are based on investment and profitability [11]:

- Factor INV: builds on the returns of a portfolio including companies with low investments less the returns of a portfolio including companies with high investment, low-minus-high INV.
- Factor ROE: builds on the returns of a portfolio including comp anies with high return-on-equity less the returns of a portfolio including companies with low return-on-equity, high-minus-low ROE.

Their model includes the market excess return (MKT) and can be recast in the following form:

$$E(r_i) = r_f + \beta^i_{MKT}E(MKT) + \beta^i_{INV}E(INV) + \beta^i_{ROE}E(ROE). \tag{18}$$

For more details about the design of the model, we refer to [11] and references therein. However their methodology is similar to the Fama and French [13] and corrected anomalies of the latter.
Even better, Fama and French propose in a recent paper [14] a five-factor model. Theoritically this new model is better than the three-factor model of Fama and French [13]. In effect, the five-factor model is directed at capturing the size, value, profitability, and investment patterns in average stock returns [14].

In sight of these versions of the APT, there is no specific factor in APT model, at least ex ante. On top of that, a first main difficulty that arises is to identify the factors for a particular stock. Identifying and quantifying each of these factors is challenging and is not a trivial endeavour. Another barrier of the APT comes from its essential assumptions regarding the existence of an arbitrage portfolio. For this reason,the model will not prevail if there is no opportunity of arbitrage in the market.

5 Conclusions

In this work, the basic ideas of the Capital Asset Pricing Model and the Arbitrage Pricing Theory are presented. Furthermore, we exhibit the practical relevance and assumptions of these models and show what make them successful for the pricing of assets. APT has an advantage over CAPM due to its flexibility, however it is more difficult in application since the factors to be used are very difficult to identify. While The CAPM emphasize efficient diversification and neglects the unsystematic risk, the APT uses the naive diversification upon the law of large numbers and therefore neglects essential risks, which is a part of the systematic risk.

In fact, the application of both methods shall be seen from a critical standpoint. As a matter of fact, the models holds relatively unrealistic assumptions of the real world. APT assumes a linear relationship between the rate of return and the risk premiums, i.e the linear relationship is insufficient it gives us a poor outcome.
The unrealistic assumptions exhibits homogeneous expectations about the assets return is intuitively contradictory. Successful investors indeed has a potential in comprehending features not accounted for. Furthermore, if all investors do think and act alike, they would likely create a 'bubble' which will inflate the asset price and reduces the risk inherent in the asset. Besides this, the betas used in the methods are extracted from historical data and are unstable through time, i.e it holds no power in explaining a certain future scenario of the market.

Finally, albeit the unrealistic assumptions of the real world, the methods in general give us an accommodating valuation in some sense. Furthermore, it is worth mentioning that no theory is perfect and it is worthwhile to learn from theory object to the criticism.
Also, in sight of the vast amount of data generated by the financial industry nowadays and the developments in Machine Learning, a natural question that arises is to initiate a bottom up reformation for the validation of some financial models. The use of techniques such as deep learning, may allow us to use hidden patterns that may be valuable in unifying models such as the CAPM and the APT to forecast risk.

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