MVE220 EXERCISE SESSION #1

2.5.5. a) A: First dice showed 3 $\{(3,1),(3,2),(3,3),(3,4),(3,5),(3,6)\}$

B: sum of dice 4 $\{(1,3),(2,2),(3,1)\}$

$$P(A|B) = \frac{P(A\cap B)}{P(B)} = \frac{1/36}{3/36} = \frac{1}{3}$$

b) A: Second dice showed 2 or less $\{(1,1),(2,1),(3,1),(4,1),(5,1),(6,1),(1,2),(2,2),(3,2),(4,2),(5,2),(6,2)\}$

B: sum of dice 4 $\{(1,3),(2,2),(3,1)\}$

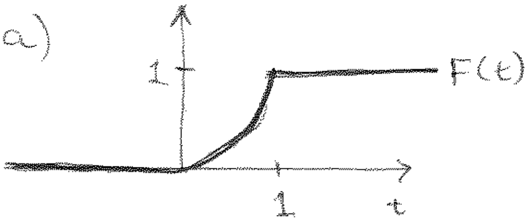
$$P(A|B) = \frac{P(A\cap B)}{P(B)} = \frac{2/36}{3/36} = \frac{2}{3}$$

c) A: Both dice showed an odd number $\{(1,1),(1,3),(1,5),(3,1),(3,3),(3,5),(5,1),(5,3),(5,5)\}$

B: sum of dice 4 $\{(1,3),(2,2),(3,1)\}$

$$P(A|B) = \frac{P(A\cap B)}{P(B)} = \frac{2/36}{3/36} = \frac{2}{3}$$

3.3.1. a)



b) $P(Y \le 0.5) = F(0.5) = 0.5^2 = 0.25$

c) $P(0.5 < Y \le 0.9) = F(0.9) - F(0.5) = 0.9^2 - 0.5^2 = 0.56$

3.5.1 $E[X] = 1\cdot 0.2 + 2\cdot 0.1 + 3\cdot 0.3 + 4\cdot 0.1 + 5\cdot 0.3 = 3.2$ //

$Var(X) = E[X^2] - E[X]^2 = 12.4 + 3.2^2 = 2.16$

$E[X^2] = 1^2\cdot 0.2 + 2^2\cdot 0.1 + 3^2\cdot 0.3 + 4^2\cdot 0.1 + 5^2\cdot 0.3 = 12.4$

$\sigma = \sqrt{Var(X)} = \sqrt{2.16} \approx 1.47$ //

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3.8.3. $E[X] = \int_0^1 \int_0^1 x f_{X,Y}(x,y)\,dy\,dx = \int_0^1 \int_0^1 x^2 + yx \,dy\,dx$

$= \int_0^1 [x^2 y + \frac{1}{2} y^2 x]_0^1 dx = \int_0^1 x^2 + \frac{1}{2} x \,dx$

$= [\frac{x^3}{3} + \frac{x^2}{4}]_0^1 = \frac{1}{3} + \frac{1}{4} = \frac{7}{12}$ //

$E[Y] = \int_0^1 \int_0^1 y f_{X,Y}(x,y)\,dy\,dx \overset{\text{symmetry}}{=} \frac{7}{12}$ //

$E[X^2] = \int_0^1 \int_0^1 x^2(x+y)\,dy\,dx = \int_0^1 [x^3 y + \frac{1}{2} y x^2]_0^1 dx$

$= \int_0^1 x^3 + \frac{1}{2} x^2 dx = [\frac{x^4}{4} + \frac{x^3}{6}]_0^1 = \frac{1}{4} + \frac{1}{6} = \frac{5}{12}$ //

$E[Y^2] = \frac{5}{12}$ //

$Var(X) = E[X^2] - E[X]^2 = \frac{5}{12} - \frac{49}{144} = \frac{11}{144}$ //

$Var(Y) = \frac{11}{144}$ //

$E[XY] = \int_0^1 \int_0^1 xy f_{XY}(x,y)\,dy\,dx = \int_0^1 \int_0^1 x^2 y + xy^2 dy\,dx$

$= \int_0^1 (\frac{1}{2} x^2 + \frac{1}{3} x)\,dx = [\frac{1}{6} x^3 + \frac{1}{6} x^2]_0^1 = \frac{1}{3}$ //

$Cov(X,Y) = E[XY] - E[X]E[Y] = \frac{1}{3} - \frac{49}{144} = -\frac{1}{144}$ //

$\rho(X,Y) = \frac{Cov(X,Y)}{\sqrt{Var(X)\,Var(Y)}} = \frac{-1/144}{\sqrt{\frac{11}{144} \cdot \frac{11}{144}}} = -\frac{1}{11}$ //

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4.3.1. a) $N(1) \sim P_o(\lambda 1) = P_o(2)$

$$P(N(1)=0) = \frac{2^0 e^{-2}}{0!} = e^{-2} \approx 0.135$$

b) $N(3) \sim P_o(3\lambda) = P_o(6)$

$$P(N(3)=4) = \frac{6^4 e^{-6}}{4!} = 54e^{-6} \approx 0.134$$

c) $N(2) \sim P_o(2\lambda) = P_o(4)$

$$P(N(2)\leq 3) = \frac{4^0 e^{-4}}{0!} + \frac{4^1 e^{-4}}{1!} + \frac{4^2 e^{-4}}{2!} + \frac{4^3 e^{-4}}{3!} \approx 0.4335$$

d) $N(0.5) \sim P_o(0.5\lambda) = P_o(1)$

$$P(N(0.5)>1) = 1 - P(N(0.5)\leq 1) = 1 - e^{-1}\left(\frac{1^0}{0!} + \frac{1^1}{1!}\right)$$
$$\approx 0.2642$$

7.2.9 $$l(\theta) = \sum_{i=1}^{9} \ln(f(x_i)) = \sum_{i=1}^{9} \ln\left(\frac{x_i}{\theta^2} e^{-x_i/\theta}\right) = \sum_{i=1}^{9}\left(\ln(x_i) - 2\ln(\theta) - \frac{x_i}{\theta}\right)$$

Now

$$0 = \frac{dl(\theta)}{d\theta} = \sum_{i=1}^{9}\left(0 - \frac{2}{\theta} + \frac{x_i}{\theta^2}\right) = -\frac{18}{\theta} + \frac{\sum_{i=1}^{9} x_i}{\theta^2}$$

$$\Rightarrow 18\theta = \sum_{i=1}^{9} x_i \Rightarrow \theta = \frac{\sum_{i=1}^{9} x_i}{18} \approx 1.317$$

7.3.1. $$p(\mu) = P(\bar{X}=0) = \frac{\mu^0 e^{-\mu}}{0!} = e^{-\mu} \leftarrow \text{strictly decreasing}$$

$$I_p = (p(2.0);\, p(0.8)) = (e^{-2.0};\, e^{-0.8}) \approx (0.135, 0.449)$$