# Value at risk

1,2,3,5,8,10,11

Given loss $L$ and confidence level $\alpha \in (0,1)$, the $100 \cdot \alpha\%$ Value at risk ($VaR_\alpha(L)$) is the $\alpha$ quantile of the distribution function

$$F_L(x) = P(L \leq x),$$

that is $VaR_\alpha(L) = F_L^{\leftarrow}(\alpha)$

generalised inverse of $F_L$

Smallest number $y$ s.t. probability of $L$ exceeding $y$ is no larger than $1-\alpha$

If $F_L(x)$ continuous, $F_L^{\leftarrow}(x) = F_L^{-1}(x)$

$\Rightarrow VaR_\alpha(L) = F_L^{-1}(\alpha)$

$\Rightarrow VaR_\alpha(L)$ is the solution $x_\alpha$ to the equation

$$F_L(x_\alpha) = \alpha.$$

Extremal index $\theta$

If $L_1, L_2, \ldots$ is a stationary stocastic process with marginal cumulative distribution function $F(x)$ and extremal index $\theta$, and $M_n = \max\{L_1, L_2, \ldots, L_n\}$

$$P(M_n \leq x) \approx F(x)^{\theta n}$$

Exercises in financial Risk - Ex3

1) PoT analysis to compute VaR for daily losses of a stock
$u=2.2$. chosen s.t. 3% of losses were above $u$
GP dist. fitted to excess over threshold.

$\sigma = 0.71, \quad \gamma = 0.15.$

Estimated 99% VaR of the stock? $\quad L \sim GP(\sigma = 0.71, \gamma = 0.15)$

$$\widehat{VaR}_{0.99} = \underset{97\%}{u} + \underbrace{VaR_{\frac{2}{3}}(L)} = 2.2 + 0.019 = 2.219.$$

$$F_L(x_\alpha) = \frac{2}{3}$$

$$1 - \left(1 + \frac{\gamma}{\sigma} x_\alpha\right)^{-1/\gamma} = \frac{2}{3}$$

$$\frac{1}{3} = \left(1 + \frac{\gamma}{\sigma} x_\alpha\right)^{-1/\gamma}$$

$$\left(\frac{1}{3}\right)^{-\gamma} - 1 = \frac{\gamma}{\sigma} x_\alpha$$

$$x_\alpha = \frac{\sigma}{\gamma}\left(3^{\gamma} - 1\right) = \frac{0.71}{0.15}\left(3^{0.15} - 1\right) = 0.019$$

2) 95% CI for shape parameter $\gamma$

$$-0.084 \pm 1.96\sqrt{0.00489} = -0.084 \pm 0.137 = (-0.221; 0.053)$$

3) Monthly losses have $GEV(\mu, \sigma, \gamma \neq 0)$ distribution. Parameters for yearly losses?

$M_{12} = \max\{X_1, \dots, X_{12}\}$

$$P(M_{12} \le x) = P(X \le x)^{12} = \exp\left\{-12\left(1 + \frac{\gamma}{\sigma}(x-\mu)\right)^{-1/\gamma}\right\}$$

$$= \exp\left\{-\left(12^{-\gamma} - 1 + 1 + 12^{-\gamma}\frac{\gamma}{\sigma}(x-\mu)\right)^{-1/\gamma}\right\}$$

$$= \exp\left\{-\left(1 + \gamma\left(\frac{12^{-\gamma} - 1}{\gamma} + \frac{1}{12^{\gamma}\sigma}(x-\mu)\right)\right)^{-1/\gamma}\right\}$$

$$= \exp\left\{-\left(1 + \frac{\gamma}{12^{\gamma}\sigma}\left(\frac{(1-12^{\gamma})\sigma}{\gamma} + x - \mu\right)\right)^{-1/\gamma}\right\}$$

$$GEV\left(\tilde{\mu} = \mu - \frac{(1-12^{\gamma})\sigma}{\gamma}, \ \tilde{\sigma} = 12^{\gamma}\sigma, \ \tilde{\gamma} = \gamma\right)$$

Exercises in Financial Risk – Ex 3

5) 7-day losses $L \sim N(-0.002, 0.03^2)$

1.7 million in a portfolio

7-day 99% VaR for the portfolio:

$$(-0.002 + z_{0.01} \cdot 0.03) \cdot 1{,}7 \cdot 10^6 = (-0.002 + 2.33 \cdot 0.03) \cdot 1{,}7 \cdot 10^6$$

$= 115.430$ SEK.

Exercises in financial risk EX3.

8. PoT VaR 95% quantile 2.60

GP fitted to excesses over threshold 2.60: $H(x) = 1 - e^{-x/\sigma}$, $\gamma = 0$

$\hat{\sigma} = 0.9$

$SD(\hat{\sigma}) = 0.057$

$\widehat{VaR}_{.8}(H)$

$VaR_{.99} = 2.60 + \widehat{VaR}_{.8}(H)$

$$H(x) = 0.8 \Rightarrow 1 - e^{-x/\sigma} = 0.8 \Rightarrow 0.2 = e^{-x/\sigma}$$

$$\Rightarrow x = -\sigma \ln(0.2) = 1.448$$

95% CI $\hat{\sigma}$: $0.9 \pm 1.96 \cdot 0.057 = 0.9 \pm 0.11172 = (\underset{\underline{\hat{\sigma}}}{0.78828}; \underset{\overline{\hat{\sigma}}}{1.01172})$

95% CI $VaR_{.8}(H)$: $(-\underline{\hat{\sigma}} \ln(0.2); -\overline{\hat{\sigma}} \ln(0.2))$

$= (1.267; 1.628)$

95% CI $VaR_{.99} = (2.60 + 1.267; 2.60 + 1.628)$

$= (3.87; 4.23)$.

Exercises in financial risk. Ex 3.

10) 5-day maximum loss (= -returns)
GEV ($\mu = 1.27$, $\sigma = 0.79$, $\gamma = 0.14$)
Extremal index $\theta = 0.73$.

Comp. One-day VaR from this.

First need % to define our VaR!

Assume 95% VaR is to be found.

$$P(M_5 \le x) = P(X \le x)^{5\theta} = (0.95)^{5\theta}$$

$$\parallel$$

$$\exp\{-(1+\frac{\gamma}{\sigma}(x-\mu))^{-1/\gamma}\}$$

$$\Rightarrow \quad x = \mu - \frac{\sigma}{\gamma}\left(1 - (-\ln(\underbrace{0.95^{5\theta}}_{=0.8293}))^{-\gamma}\right)$$

$$= 2.76.$$

11) PoT to compute VaR. Threshold $u = 1.7$ chosen s.t. 5% of losses above $u$. Fit GP dist to excesses L

$\Rightarrow \sigma = 0.8$, $\gamma = 0.1$.

Find estimate of the 99% VaR.

$$VaR_{0.99} = u + \underbrace{VaR_{0.8}(L)}_{} = 3.097$$

$$F_L(x_\alpha) = 0.8$$

$$1 - (1 + \frac{\gamma}{\sigma} x_\alpha)^{-1/\gamma} = 0.8$$

$$(1 + \frac{\gamma}{\sigma} x_\alpha)^{-1/\gamma} = 0.2$$

$$x_\alpha = \frac{\sigma}{\gamma}(0.2^{-\gamma} - 1) = 1.397$$