Exercises in Financial Risk: Extreme Value Statistics
March 16, 2018

1 In a peaks over thresholds analysis to compute VaR for the daily losses of a stock, the threshold $u = 2.2$ was chosen such that 3% of the losses were above u, and one then fitted a GP distribution to the excesses over the threshold. The estimated scale parameter was $\sigma = 0.71$ and the estimated shape parameter was $\gamma = 0.15$. What is then the estimated 99% daily VaR for the stock?

2 Below are shown the maximum likelihood estimates and the estimates of the covariance matrix from the block maxima method applied to a data set. Compute a 95% confidence interval for the shape parameter ξ.

data. This leads to the maximum likelihood estimate

$$(\hat{\mu}, \hat{\sigma}, \hat{\xi}) = (-1.64, 0.27, -0.084),$$

with a maximized value of the log-likelihood equal to -14.3. The corresponding estimated variance-covariance matrix is

$$V = \begin{bmatrix} 0.00141 & 0.000214 & -0.000795 \\ 0.000214 & 0.000652 & -0.000441 \\ -0.000795 & -0.0000441 & 0.00489 \end{bmatrix}.$$

3 Suppose that the monthly maximum losses of a financial instrument has a GEV distribution with location parameter μ, scale parameter σ and shape parameter $\gamma \neq 0$, and that the monthly losses are independent. Then also the yearly maximum losses have a GEV distribution. What are the parameters of this distribution?

4 Suppose $X_1, X_2, \ldots$ are independent and exponentially distributed random variables with parameter σ (so that their cumulative distribution function is $F(x) = 1 - e^{-x/\sigma}$ for $0 \leq x$ and $F(x) = 0$ for $x < 0$), and define $M_n = \max\{X_1, \ldots X_n\}$. Find norming values a_n, b_n such that

$$P(\frac{M_n - a_n}{b_n}) \to e^{-e^{-x}}, \quad \text{as } n \to \infty.$$

5 Suppose a data set contains 7-day losses (= - returns) during the last 2 years for a portfolio, and that the mean and standard deviation of the losses are -0.002 and 0.03, respectively. If one assumes that losses are normally distributed and that SEK 1.7 million is invested in the portfolio, what is then the 7-day 99% VaR, in SEK, for the portfolio?

6 Suppose that daily portfolio losses are identically distributed and that 7-day maximum losses expressed in % have a GEV distribution function with estimated location parameter $\hat{\mu} = 0.96$, estimated scale parameter $\hat{\sigma} = 1.17$, and estimated shape parameter $\hat{\gamma} = 0.21$, and that the extremal index has been estimated to be $\hat{\theta} = 0.73$. Compute an estimate for the 1-day 95% VaR for such a portfolio with an initial value of SEK 1.7 million.

7 Suppose that an excesses of a threshold u has a GP distribution with scale parameter σ and shape parameter γ, where it for simplicity is assumed that $\gamma > 0$. Assume that $v > 0$. Then the conditional distribution of excesses of the level $u + v$ is also a GP distribution. What are the parameters of this distribution?

8 In a peaks over thresholds analysis to compute VaR one assumed that from very long experience it was known that the 95% quantile of the losses was exactly 2.60, with negligible error. Further, one used a GP distribution with shape parameter $\gamma = 0$, so that the distribution function was $H(x) = 1 - e^{-x/\sigma}$, to model the excesses over the threshold 2.60. The estimated parameter was $\hat{\sigma} = 0.90$, and the standard deviation of $\hat{\sigma}$ was estimated to be 0.057.

Let $\widehat{VaR}_{.8}(H)$ be the the estimated 80% VaR of the distribution function H, so that the estimated 0.99 VaR of the losses equals $2.60 + \widehat{VaR}_{.8}(H)$. Then a 95% confidence interval for the 99% VaR of the losses is

9 Suppose that 5-day maximum losses are mutually independent and have a GEV distribution function with location parameter μ, scale parameter σ, and shape parameter γ. Then 20-day maximum losses also have a GEV distribution. What are the location, scale, and shape parameter for this distribution?

10 Suppose that the 5-day maximum loss (= - returns) has a GEV distribution function with location parameter $\mu = 1.27$, scale parameter $\sigma = 0.79$ and shape parameter $\gamma = 0.14$, and that additionally the extremal index is $\theta = 0.73$. Compute the one-day VaR can be computed from this.

11 In a peaks over thresholds analysis to compute VaR one used the threshold $u = 1.7$ chosen such that 5% of the losses were above u and fitted a GP distribution to the excesses over the thresholds. The estimated

parameters were $\sigma = 0.8$ and $\gamma = 0.1$. Find an estimate of the 99% VaR.

12 Suppose $X_1, X_2, \ldots$ are independent and uniformly distributed random variables (so that their cumulative distribution function is $F(x) = x$ for $x \in [0, 1]$), and define $M_n = \max\{X_1, \ldots X_n\}$. Show that for normalizing constants $a_n = 1, b_n = \frac{1}{n}$ it holds that

$$P(\frac{M_n - a_n}{b_n} \leq x) \rightarrow e^x, \quad \text{for } x \leq 0, \quad \text{as } n \rightarrow \infty.$$

13 Let X_1 and X_2 be independent random variables which both have a GEV distribution function with shape parameter $\gamma = 0$, scale parameter $\sigma = 1.3$ and location parameter $\mu = 2.1$, so that they both have cumulative distribution function

$$G(x) = e^{-e^{-\frac{x-2.1}{1.3}}}.$$

Then also $M = \max X_1, X_2$, the maximum of X_1 and X_2, has a GEV distribution function. Find the shape, scale, and location parameters of this GEV distribution function of M.

14 Below are shown the maximum likelihood estimates and the estimates of the covariance matrix from the block maxima method applied to a data set on glass fibre strengths. Find a 95% confidence interval for the scale parameter σ.

data. This leads to the maximum likelihood estimate

$$(\hat{\mu}, \hat{\sigma}, \hat{\xi}) = (-1.64, 0.27, -0.084),$$

with a maximized value of the log-likelihood equal to -14.3. The corresponding estimated variance-covariance matrix is

$$V = \begin{bmatrix} 0.00141 & 0.000214 & -0.000795 \\ 0.000214 & 0.000652 & -0.000441 \\ -0.000795 & -0.0000441 & 0.00489 \end{bmatrix}.$$