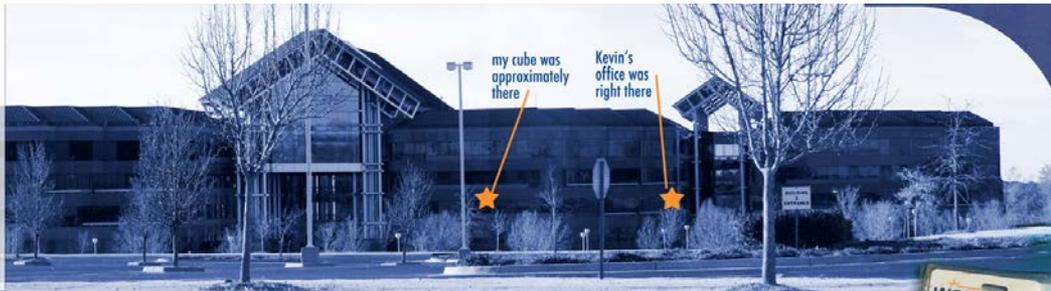


HOME TO THE  
LARGEST  
MULTI-BILLION  
DOLLAR  
CORPORATE  
ACCOUNTING  
FRAUD IN  
THE HISTORY  
OF OUR NATION



CLINTON, MISSISSIPPI -- FORMER CORPORATE WORLD HEADQUARTERS FOR WORLDCOM

# As the **WORLDCOM** turns.

CORPORATE CRIME, FBI INVESTIGATIONS, MASS LAYOFFS, LOSSES, MERGERS AND MORE...

## MY JOB DESCRIPTION:

I started working at WorldCom in September of 1999. While I was there, I was the designer for their Corporate Intranet, which happened to be the busiest intranet site in the world. I also did graphics for the internet site as well. I quit my job soon after Quinn was born in February of 2002.



## BITTER:

Well, we lost it all! Kevin got laid off after working there for over 8 years!!! We had just bought a house, and had a baby!! Not to mention I had just quit my job (at WorldCom) to be a stay-at-home mom! We both lost our ENTIRE 401k's and Kevin also lost hundreds of thousands of dollars in vested stock options. Times were tough!

## SWEET:

This is where we MET!!! This is where we worked when we got married and had our first child! This is where our life together began. So even though we lost all of our money, we gained ALL of those things that money just CANNOT BUY!! Like true love, a best friend, and family! I wouldn't trade this WorldCom experience for anything!!!!

## KEVIN'S JOB DESCRIPTION:

Kevin started working at WorldCom when it was actually still LDDS. It's hard to just sum up ALL of what he did for them. He was there through over 60 mergers and acquisitions and played a major part in integrating the systems of all of the new companies. He was also over all of their Corporate Internet and Intranet Systems. Kevin was laid off from WorldCom in April of 2002, which was 2 months after I quit.

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“As an alternative to the traditional 30-year mortgage, we also offer an interest-only mortgage, balloon mortgage, reverse mortgage, upside down mortgage, inside out mortgage, loop-de-loop mortgage, and the spinning double axel mortgage with a triple lutz.”



Gudrun January 2005

326 MEuro loss

72 % due to forest losses

4 times larger than second largest

## **This course:**

- Learn some fun things about financial risks
- Learn some basic risk management tools from Extreme Value Statistics
- See some basic quantitative Credit Risk models

### ***But not***

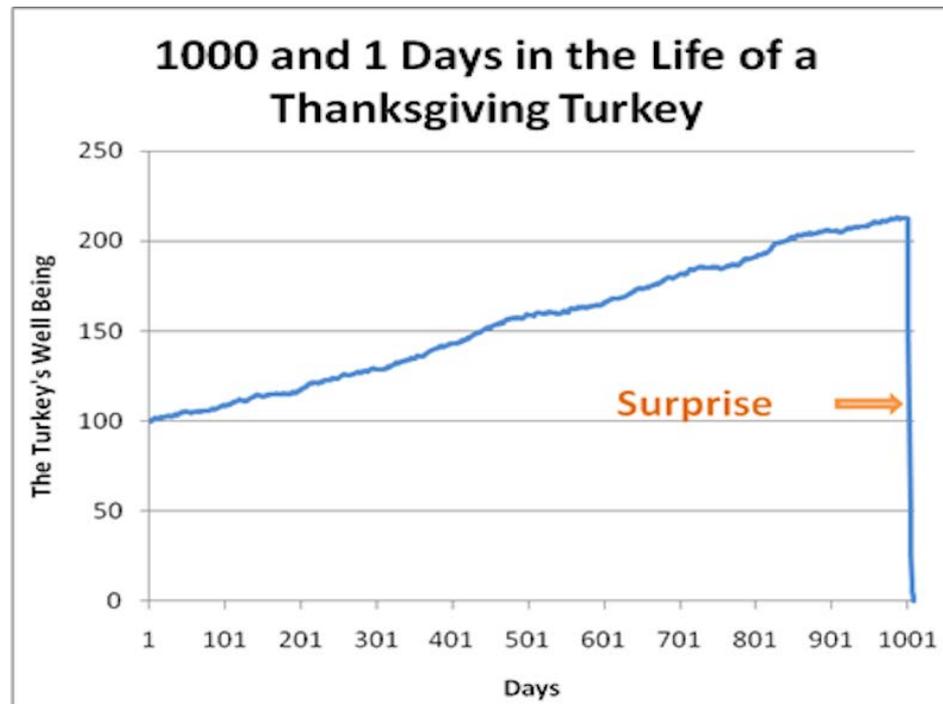
- A complete systematic account of financial risk management
- Financial time series modeling
- Black-Scholes option pricing methods
- Macroeconomics

### ***And***

**There are risks which cannot be handled by mathematical models!!**

Risk: event or action which prevents an institution from meeting its obligations or reaching its goals.

If one does not understand the real-world situation well enough, the best quantitative tools will not help. Taleb's Turkey example:



## Model checking and adjustment

- Independence
- Stationarity
- Distributional fit
- Period and threshold choice

Statistical model



Data analysis

- Choice of distributions
- Estimation (often of quantiles)
- Confidence intervals
- “Prediction intervals”
- Hypothesis testing (*not this course*)
- Regression and dependence modeling (*not this course*)
- Prediction (*not this course*)
- ***Understanding!!***

# Refresh your basic statistics knowledge!

- Distribution functions
- Independent events, conditional probabilities
- Poisson process
- Expected value, variance, moments
- Correlation
- Point estimation
- Confidence intervals
- qq-plots, see <http://data.library.virginia.edu/understanding-q-q-plots/>  
(*R* is a free software environment for statistical computing and graphics)

- Credit risk
- Market risk
- Operational risk
- Insurance risk
- Liquidity risk
- Reputational risk
- Legal risk
- and so on ...



*Market risk:* risk that the value of a portfolio changes due to changes of market prices, exchange rates etc.

*Credit risk:* risk that the value of a portfolio changes because a debtor cannot meet his obligations.

*Operational risk:* risk caused by problems in internal processes, people, systems

Basel III: A global, voluntary regulatory framework on bank capital adequacy, stress testing, and market liquidity risk

Solvency 2: A Directive in European Union law that codifies and harmonises the EU insurance regulation. Primarily this concerns the amount of capital that EU insurance companies must hold to reduce the risk of insolvency.

## Risk factors

$L = \text{"-P/L"} = \text{Loss} - \text{Profit} = F(X_1, \dots, X_d)$ ,  $X_1, \dots, X_d$ , risk factors, e.g. exchange rates, interest rates, index movements, stock prices, ....

**Example:** Linear portfolio,  $\alpha_i$  # shares of stock  $i$ , stock price  $S_{t,i}$

$$L = - \sum_{i=1}^d \alpha_i S_{t+1,i} + \sum_{i=1}^d \alpha_i S_{t,i} = \sum_{i=1}^d \alpha_i S_{t,i} \left( \frac{S_{t+1,i} - S_{t,i}}{S_{t,i}} \right)$$

# How big is the risk?

## Quantitative risk management methods:

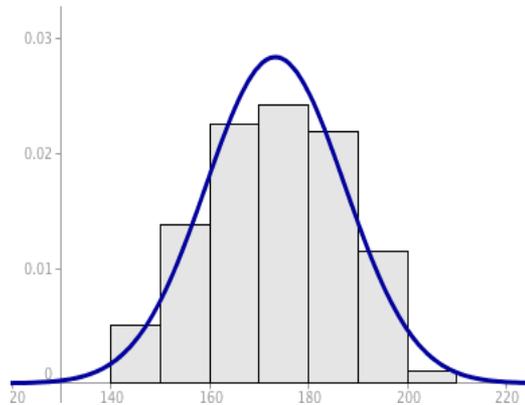
- Historical data or historical simulation
- Stress testing (“scenarios”)
- Sensitivity measures (“the greeks”)
- Full statistical modeling (often multivariate normal + linear portfolio)
- Semiparametric modeling of the “tails” of the *loss-profit* distribution (univariate Extreme Value Statistics) (*this course*)
- Semiparametric modeling of the tails of the multivariate distribution of the risk factors (multivariate Extreme Value Statistics, “Copulas”) + computation of the *loss-profit* distribution analytically or via stochastic simulation

# How big is the risk?

**Mathematics** → shapes of possible risk distributions

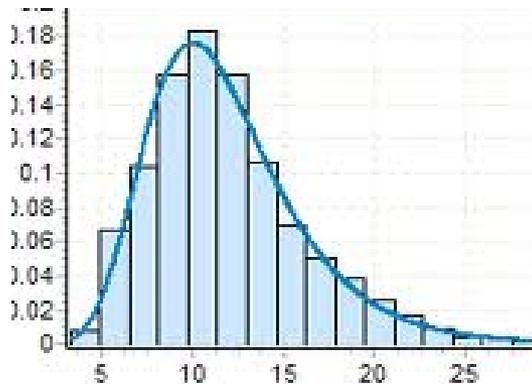
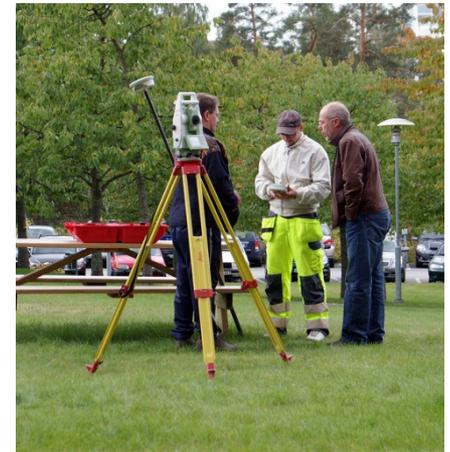
**Statistics** → choice of specific risk distribution (with uncertainty)

**Risk distribution** → quantifies risk, perhaps via VaR or ES, perhaps multivariate



Normal distribution

Error in the measured distance = sum of many small measurement errors



Generalized Extreme Value distribution

Highest water level during year = maximum of daily water levels



## **Basic EVS:**

**--- Block Maxima: GEV distribution for maxima**

**--- Peaks over Thresholds: GP distribution for tails**

Why?

- **stability:** maxima of variables which are GEV distributed are also GEV; going to higher levels preserves the GP distribution of exceedances (cf. “standard statistics: sums of normally distributed variables have a normal distribution”)
- **asymptotics:** maxima of many independent variables are often (approximately) GEV distributed; asymptotically tails are GP when maxima are EV (cf. “standard statistics: sums of many small “errors” are often (approximately) normally distributed the “central limit theorem”)
- **“transition”:** easy to go back and forth between GP and GEV

**but don't believe in models blindly**

## The Block Maxima method (Coles p. 45-53)

the Generalized Extreme Value (GEV) distribution:

$$G(x) = \exp\left\{-\left(1 + \gamma \frac{x - \mu}{\sigma}\right)^{-1/\gamma}\right\}$$

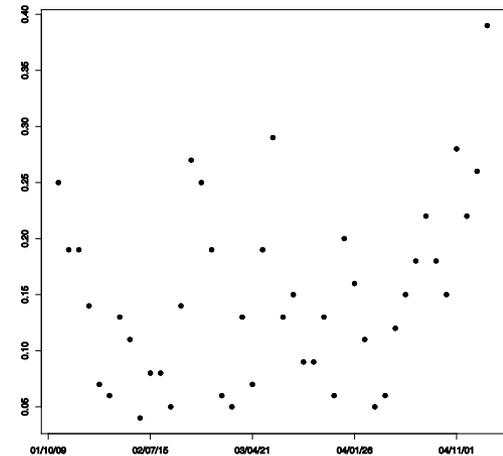
the special case  $\gamma = 0$  is the Gumbel distribution

$$G(x) = \exp\left\{-e^{-\frac{x - \mu}{\sigma}}\right\}$$

Fit to measured block (=weekly, or monthly, or ...) maxima, assuming independence, using maximum likelihood. Find  $p$ -th quantile  $\hat{x}_p$  (= value such that the risk that it is exceeded is  $100(1 - p)\%$ ) by solving

$$\hat{G}(\hat{x}_p) = p$$

where  $\hat{\phantom{x}}$  means that parameters are replaced by their estimated values. Confidence intervals  $\rightarrow$  Later lectures



Maximum long term US interest rate, constant maturity, nominal 1 month, percentage points

## Some mathematics behind the Block Maxima Method:

$X_1, X_2, \dots$  independent identically distributed (i.i.d.) random variables with distribution function (d.f.)  $F$

$M_n = \max\{X_1, \dots, X_n\}$  maximum of the first  $n$  variables (e.g.  $n = 30$  and  $X$  daily interest rates gives plot on previous slide)

$$\begin{aligned} P(M_n \leq x) &= P(X_1 \leq x, \dots, X_n \leq x) \\ &= P(X_1 \leq x) \times \dots \times P(X_n \leq x) = F(x)^n \end{aligned}$$

**Exercise:** Show that the GEV distributions are *max-stable*, i.e. that the maximum of  $n$  i.i.d. GEV-distributed variables also

have an GEV distribution, i.e. that if  $G(x) = \exp\left\{-\left(1 + \gamma \frac{x - \mu}{\sigma}\right)^{-1/\gamma}\right\}$  then there are  $\mu_n, \sigma_n$  such that

$$G(x)^n = \exp\left\{-\left(1 + \gamma \frac{x - \mu_n}{\sigma_n}\right)^{-1/\gamma}\right\}$$

and find  $\mu_n, \sigma_n$ .

**Theorem:** *The distribution function  $G$  is max-stable if and only if it is an GEV distribution*

In the previous exercise it was shown that the EV distributions are max-stable, which proves half of this theorem. The other half consists of solving the functional equations

$$G(x)^n = G\left(\frac{x - \mu_n}{\sigma_n}\right), \text{ for } n = 1, 2, \dots$$

to find that the GEV distributions are the only solutions.

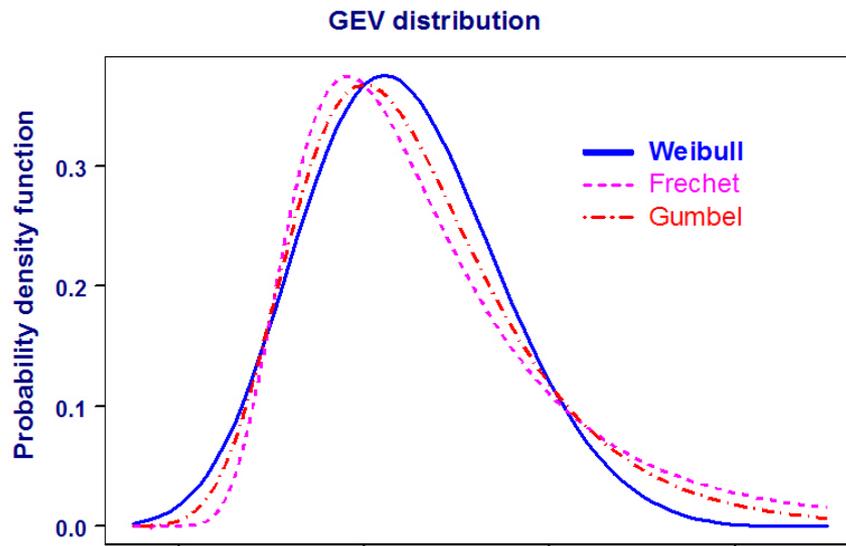
**Theorem:** *If there are constants  $b_n > 0, a_n$  such that*

$$P\left(\frac{M_n - a_n}{b_n} \leq x\right) \rightarrow G(x), \text{ as } n \rightarrow \infty \text{ for all } x$$

*then  $G(x)$  is an GEV distribution. (cf. the central limit theorem)*

This is proved by showing that it follows that  $G(x)$  must be max-stable

**Challenge:** Prove this!



$$g(x) = \frac{d}{dx}G(x) = \frac{1}{\sigma} \left(1 + \gamma \frac{x-\mu}{\sigma}\right)_+^{-1/\gamma-1} \exp\left\{-\left(1 + \gamma \frac{x-\mu}{\sigma}\right)_+^{-1/\gamma}\right\}$$

Densities of the Generalized Extreme Value distribution

$\gamma > 0$  Frechet distribution, finite left endpoint of distribution:  $x > \mu + \frac{\sigma}{\gamma}$

$\gamma = 0$  Gumbel distribution, unbounded support

$\gamma < 0$  Weibull distribution, finite right endpoint of distribution:  $x < \mu + \frac{\sigma}{|\gamma|}$

the distribution is “heavilytailed” for  $\gamma > 0$ : then moments of order greater than  $1/\gamma$  are infinite/don’t exist; thus if  $\gamma > 1/2$  then the variance doesn’t exist, if  $\gamma > 1$  then the mean doesn’t exist either

**Exercise:** Suppose  $X_1, X_2, \dots$  are i.i.d. and Pareto distributed random variables with distribution function (d.f.)

$$F(x) = 1 - \left(\frac{K}{x}\right)^\alpha, \quad x \geq K, \quad K, \alpha > 0.$$

Show that there is a d.f.  $G(x)$  such that

$$P\left(\frac{M_n - a_n}{b_n} \leq x\right) \rightarrow G(x) \text{ as } n \rightarrow \infty$$

for  $a_n = Kn^{1/\alpha}$ ,  $b_n = Kn^{1/\alpha}$ , and find  $G$ .

*Hint:* Use  $\left(1 + \frac{a}{n}\right)^n \rightarrow e^a$  as  $n \rightarrow \infty$ , and that

$$P\left(\frac{M_n - a_n}{b_n} \leq x\right) = P(M_n \leq b_n x + a_n)$$

(A perhaps unnecessary explanation) **What does**

$$P\left(\frac{M_n - a_n}{b_n} \leq x\right) \rightarrow G(x), \text{ as } n \rightarrow \infty \text{ for all } x,$$

**mean in practice?** That  $P\left(\frac{M_n - a_n}{b_n} \leq x\right) \approx G(x)$ , for large  $n$ , or,

with  $y = b_n x + a_n$  and  $G(x) = \exp\left\{-\left(1 + \gamma \frac{x - \mu'}{\sigma'}\right)^{-1/\gamma}\right\}$ , that

$$\begin{aligned} P(M_n \leq y) &\approx G\left(\frac{y - a_n}{b_n}\right) = \exp\left\{-\left(1 + \gamma \frac{y - (a_n + b_n \mu')}{b_n \sigma'}\right)^{-1/\gamma}\right\} \\ &= \exp\left\{-\left(1 + \gamma \frac{y - \mu}{\sigma}\right)^{-1/\gamma}\right\}, \text{ for } \mu = a_n + b_n \mu', \sigma = b_n \sigma'. \end{aligned}$$

Since all the parameters are unknown anyway, we are left with the problem of estimating  $\mu, \sigma$  from data, *i.e.* with the Block Maxima method.

