Financial Risk
4-rd quarter 2017/18

"As an alternative to the traditional 30-year mortgage, we also offer an interest-only mortgage, balloon mortgage, reverse mortgage, upside down mortgage, inside out mortgage, loop-de-loop mortgage, and the spinning double axel mortgage with a triple lutz."

Gudrun January 2005
326 MEuro loss
72 % due to forest losses
4 times larger than second largest

This course:

- Learn some fun things about financial risks
- Learn some basic risk management tools from Extreme Value Statistics
- See some basic quantitative Credit Risk models

But not

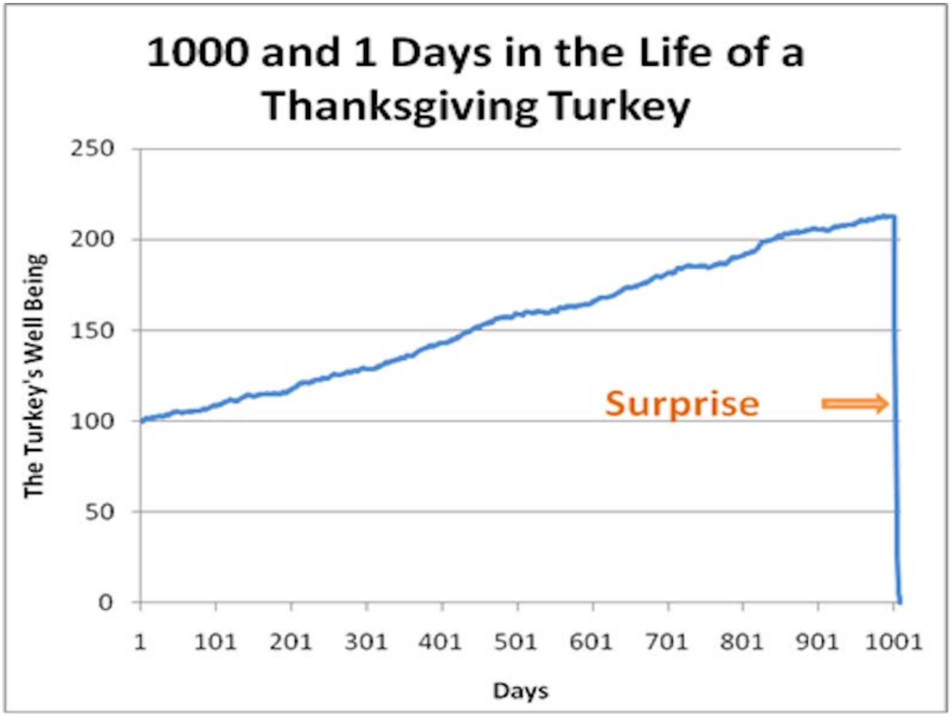
- A complete systematic account of financial risk management
- Financial time series modeling
- Black-Scholes option pricing methods
- Macroeconomics

And

There are risks which cannot be handled by mathematical models!!

Risk: event or action which prevents an institution from meeting its obligations or reaching its goals.

If one does not understand the real-world situation well enough, the best quantitative tools will not help. Taleb's Turkey example:



Model checking and adjustment

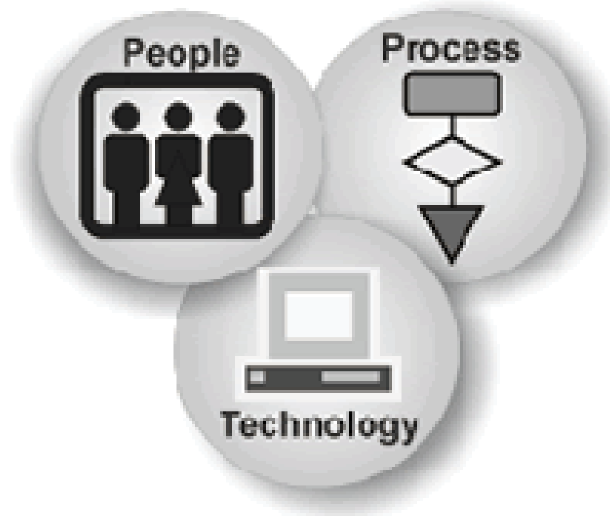
- Independence
- Stationarity
- Distributional fit
- Period and threshold choice

- Choice of distributions
- Estimation (often of quantiles)
- Confidence intervals
- “Prediction intervals”
- Hypothesis testing ***(not this course)***
- Regression and dependence modeling ***(not this course)***
- Prediction ***(not this course)***
- ***Understanding!!***

Refresh your basic statistics knowledge!

- Distribution functions
- Independent events, conditional probabilities
- Poisson process
- Expected value, variance, moments
- Correlation
- Point estimation
- Confidence intervals
- qq-plots, see http://data.library.virginia.edu/understanding-q-q-plots/ (*R* is a free software environment for statistical computing and graphics)

- Credit risk
- Market risk
- Operational risk
- Insurance risk
- Liquidity risk
- Reputational risk
- Legal risk
- and so on …



Operational risk

Market risk: risk that the value of a portfolio changes due to changes of market prices, exchange rates etc.

Credit risk: risk that the value of a portfolio changes because a debtor cannot meet his obligations.

Operational risk: risk caused by problems in internal processes, people, systems

Basel III: A global, voluntary regulatory framework on bank capital adequacy, stress testing, and market liquidity risk

Solvency 2: A Directive in European Union law that codifies and harmonises the EU insurance regulation. Primarily this concerns the amount of capital that EU insurance companies must hold to reduce the risk of insolvency.

Risk factors

L= "-P/L" = Loss – Profit =$F(X_1, \ldots X_d)$, $X_1, \ldots X_d$, risk factors, *e.g.* exchange rates, interest rates, index movements, stock prices, ….

Example: Linear portfolio, α_i # shares of stock *i*, stock price $S_{t,i}$

$$L = -\sum_{i=1}^{d} \alpha_i S_{t+1,i} + \sum_{i=1}^{d} \alpha_i S_{t,i} = \sum_{i=1}^{d} \alpha_i S_{t,i} \left(\frac{S_{t+1,i} - S_{t,i}}{S_{t,i}} \right)$$

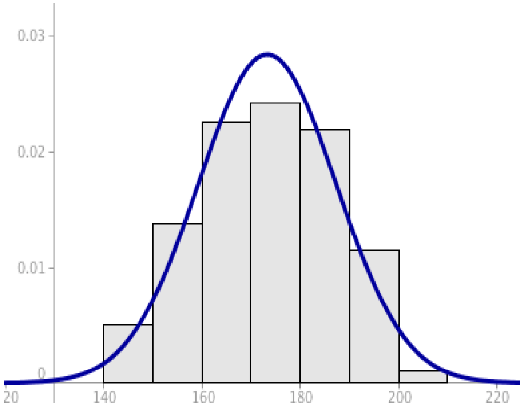
How big is the risk?

Quantitative risk management methods:

- Historical data or historical simulation
- Stress testing ("scenarios")
- Sensitivity measures ("the greeks")
- Full statistical modeling (often multivariate normal + linear portfolio)
- Semiparametric modeling of the "tails" of the *loss-profit* distribution (univariate Extreme Value Statistics) ***(this course)***
- Semiparametric modeling of the tails of the multivariate distribution of the risk factors (multivariate Extreme Value Statistics, "Copulas") + computation of the *loss-profit* distribution analytically or via stochastic simulation

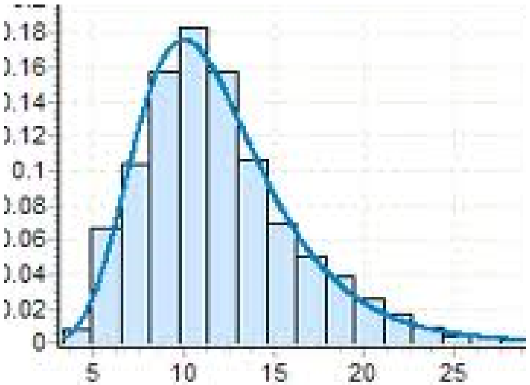
How big is the risk?

Matematics → shapes of possible risk distributions
Statistics → choice of specific risk distribution (with uncertainty)
Risk distribution → quantifies risk, perhaps via VaR or ES, perhaps multivariate



Normal distribution

Error in the measured distance = sum of many small measurement errors



Generalized Extreme Value distribution

Highest water level during year = maximum of daily water levels

Basic EVS:

--- Block Maxima: GEV distribution for maxima

--- Peaks over Thresholds: GP distribution for tails

Why?

- **stability:** maxima of variables which are GEV distributed are also GEV; going to higher levels preserves the GP distribution of exceedances (cf. "standard statistics: sums of normally distributed variables have a normal distribution)

- **asymptotics:** maxima of many independent variables are often (approximately) GEV distributed; asymptotically tails are GP when maxima are EV (cf. "standard statistics: sums of many small "errors" are often (approximately) normally distributed the "central limit theorem")

- **"transition":** easy to go back and forth between GP and GEV

but don't believe in models blindly

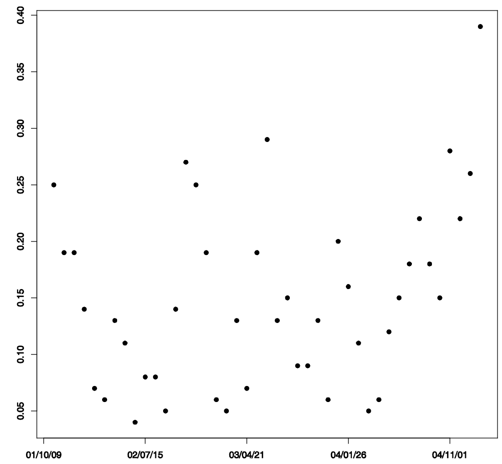
The Block Maxima method *(Coles p. 45-53)*

the Generalized Extreme Value (GEV) distribution:

$$G(x) = \exp\{-\left(1 + \gamma \frac{x-\mu}{\sigma}\right)^{-1/\gamma}\}$$

the special case $\gamma = 0$ is the Gumbel distribution

$$G(x) = \exp\{-e^{-\frac{x-\mu}{\sigma}}\}$$



Maximum long term US interest rate, constant maturity, nominal 1 month, percentage points

Fit to measured block (=weekly, or monthly, or …) maxima, assuming independence, using maximum likelihood. Find *p*-th quantile $\hat{x}_p$ (= value such that the risk that it is exceeded is $100(1-p)\%$) by solving

$$\hat{G}(\hat{x}_p) = p$$

where $\widehat{\ }$ means that parameters are replaced by their estimated values. Confidence intervals → Later lectures

Some mathematics behind the Block Maxima Method:

$X_1, X_2, \ldots$ independent identically distributed (i.i.d.) random variables with distribution function (d.f.) F

$\mathrm{M_n} = \max\{X_1, \ldots, X_n\}$ maximum of the first n variables (*e.g. n* = 30 and X daily interest rates gives plot on previous slide)

$$P(M_n \le x) = P(X_1 \le x, \ldots X_n \le x)$$
$$= P(X_1 \le x) \times \cdots \times P(X_1 \le x) = F(x)^n$$

Exercise: Show that the GEV distributions are *max-stable, i.e.* that the maximum of n i.i.d. GEV-distributed variables also have an GEV distribution, *i.e.* that if $G(x) = \exp\{-\left(1 + \gamma \frac{x-\mu}{\sigma}\right)^{-1/\gamma}\}$ then there are μ_n, σ_n such that

$$G(x)^n = \exp\{-\left(1 + \gamma \frac{x - \mu_n}{\sigma_n}\right)^{-1/\gamma}$$

and find μ_n, σ_n.

Theorem: *The distribution function* G *is max-stable if and only if it is an GEV distribution*

In the previous exercise it was shown that the EV distributions are max-stable, which proves half of this theorem. The other half consists of solving the functional equations

$$G(x)^n = G(\tfrac{x-\mu_n}{\sigma_n}), \text{ for } n = 1, 2, \ldots$$

to find that the GEV distributions are the only solutions.

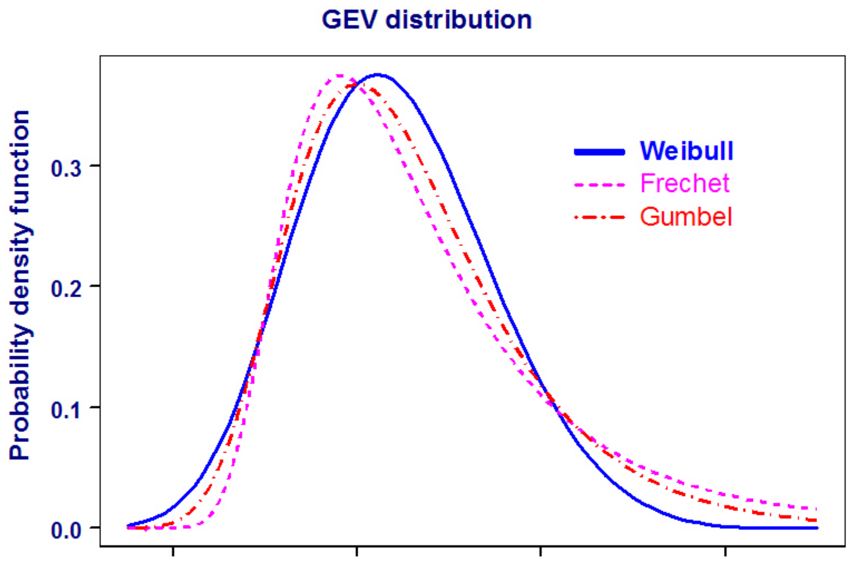
Theorem: *If there are constants* $b_n > 0, a_n$ *such that*

$$P\left(\tfrac{M_n - a_n}{b_n} \leq x\right) \to G(x), \text{ as } n \to \infty \text{ for all } x$$

then G(x) is an GEV distribution. ***(cf. the central limit theorem)***

This is proved by showing that it follows that *G(x)* must be max-stable

Challenge: Prove this!



$$g(x) = \frac{d}{dx}G(x) = \frac{1}{\sigma}(1+\gamma\frac{x-\mu}{\sigma})_+^{-1/\gamma-1} \exp\{-(1+\gamma\frac{x-\mu}{\sigma})_+^{-1/\gamma}\}$$

Densities of the Generalized Extreme Value distribution

$\gamma > 0$ Frechet distribution, finite left endpoint of distribution: $x > \mu + \frac{\sigma}{\gamma}$

$\gamma = 0$ Gumbel distribution, unbounded support

$\gamma < 0$ Weibull distribution, finite right endpoint of distribution: $x < \mu + \frac{\sigma}{|\gamma|}$

the distribution is "heavytailed" for $\gamma > 0$: then moments of order greater than $1/\gamma$ are infinite/don't exist; thus if $\gamma > 1/2$ then the variance doesn't exist, if $\gamma > 1$ then the mean doesn't exist either

Exercise: Suppose $X_1, X_2, \ldots$ are i.i.d. and Pareto distributed random variables with distribution function (d.f.)

$$F(x) = 1 - \left(\frac{K}{x}\right)^{\alpha}, \qquad x \geq K, \qquad K, \alpha > 0.$$

Show that there is a d.f. $G(x)$ such that

$$P\left(\frac{M_n - a_n}{b_n} \leq x\right) \rightarrow G(x) \quad as\ n \rightarrow \infty$$

for $a_n = Kn^{1/\alpha}, b_n = Kn^{1/\alpha}$, and find G.

Hint: Use $\left(1 + \frac{a}{n}\right)^n \rightarrow e^a \ \ as\ n \rightarrow \infty$, and that

$$P\left(\frac{M_n - a_n}{b_n} \leq x\right) = P(M_n \leq b_n x + a_n)$$

(A perhaps unnecessary explanation) **What does**

$$P(\frac{M_n - a_n}{b_n} \leq x) \to G(x), \text{ as } n \to \infty \text{ for all } x,$$

mean in practice? That $P(\frac{M_n - a_n}{b_n} \leq x) \approx G(x)$, for large n, or,

with $y = b_n x + a_n$ and $G(x) = \exp\{-(1+\gamma\frac{x-\mu'}{\sigma'}))^{-1/\gamma}\}$, that

$$\begin{aligned} P(M_n \leq y) &\approx G(\frac{y-a_n}{b_n}) = \exp\{-(1+\gamma\frac{y-(a_n+b_n\mu')}{b_n\sigma'})^{-1/\gamma}\} \\ &= \exp\{-(1+\gamma\frac{y-\mu}{\sigma})^{-1/\gamma}\}, \text{ for } \mu = a_n + b_n\mu', \ \sigma = b_n\sigma. \end{aligned}$$

Since all the parameters are unknown anyway, we are left with the problem of estimating $\mu, \ \sigma$ from data, *i.e.* with the Block Maxima method.