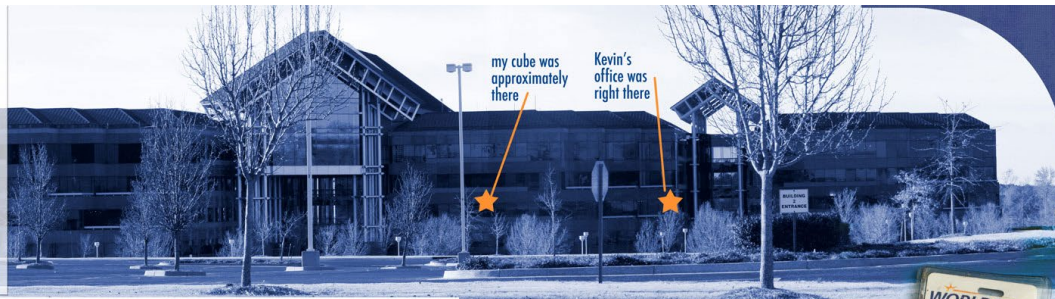


HOME TO THE
LARGEST
MULTI-BILLION
DOLLAR
CORPORATE
ACCOUNTING
FRAUD IN
THE HISTORY
OF OUR NATION



CLINTON, MISSISSIPPI -- FORMER CORPORATE WORLD HEADQUARTERS FOR WORLDCOM

As the **WORLDCOM** turns.

CORPORATE CRIME, FBI INVESTIGATIONS, MASS LAYOFFS, LOSSES, MERGERS AND MORE...

BITTER: Well, we lost it all! Kevin got laid off after working there for over 8 years!!! We had just bought a house, and had a baby!! Not to mention I had just quit my job (at WorldCom) to be a stay-at-home-mom! We both lost our ENTIRE 401K's and Kevin also lost hundreds of thousands of dollars in vested stock options. Times were tough!

SWEET: This is where we MET!!! This is where we worked when we got married and had our first child! This is where our life together began. So even though we lost all of our money, we gained ALL of those things that money just CANNOT BUY!! Like true love, a best friend, and family! I wouldn't trade this WorldCom experience for anything!!!!

KEVIN'S JOB DESCRIPTION:

Kevin started working at WorldCom when it was actually still LDDS. It's hard to just sum up ALL of what he did for them. He was there through over 60 mergers and acquisitions and played a major part in integrating the systems of all of the new companies. He was also over all of their Corporate Internet and Intranet Systems. Kevin was laid off from WorldCom in April of 2002, which was 2 months after I quit.

MY JOB DESCRIPTION:

I started working at WorldCom in September of 1999. While I was there, I was the designer for their Corporate Intranet, which happened to be the busiest intranet site in the world. I also did graphics for the internet site as well. I also quit my job soon after Quinn was born in February of 2002.



Financial Risk 4-rd quarter 2018/19 Lecture 2

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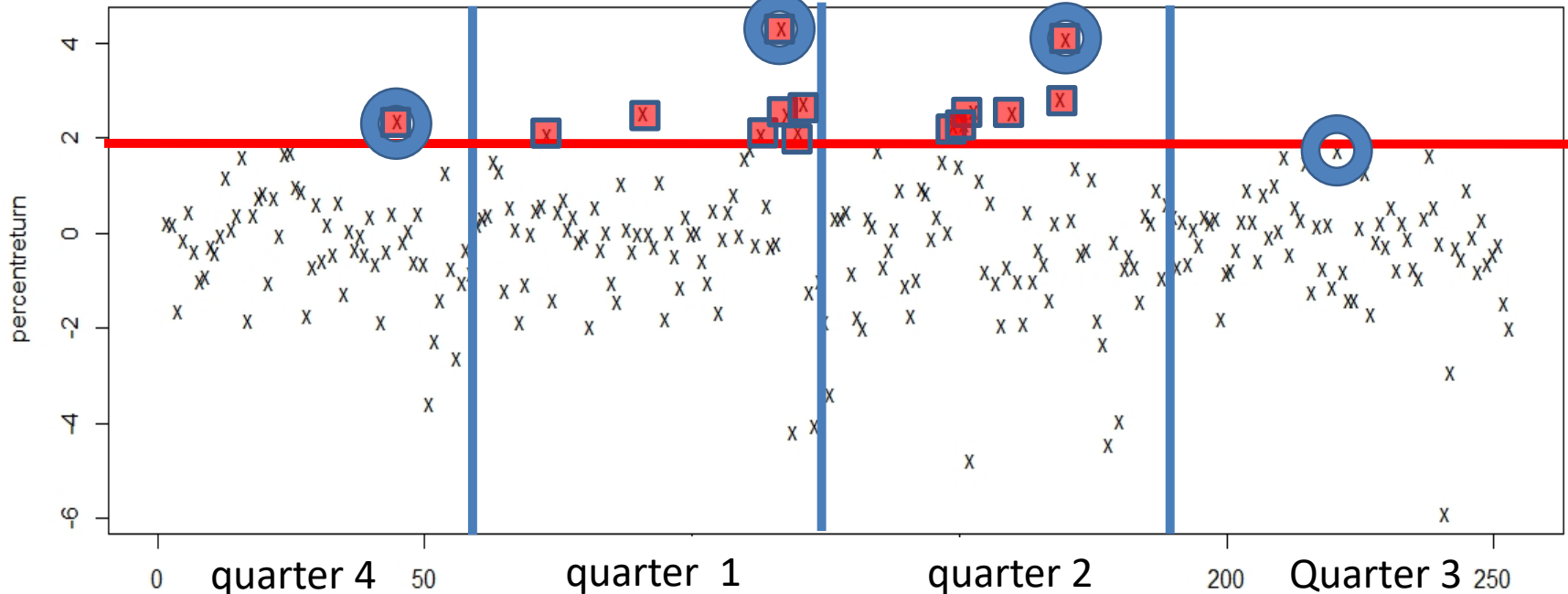


“As an alternative to the traditional 30-year mortgage, we also offer an interest-only mortgage, balloon mortgage, reverse mortgage, upside down mortgage, inside out mortgage, loop-de-loop mortgage, and the spinning double axel mortgage with a triple lutz.”



Gudrun January 2005
326 MEuro loss
72 % due to forest losses
4 times larger than second largest

Apple losses ($= -100 \times \frac{\text{price tomorrow} - \text{price today}}{\text{price today}}$) one year back

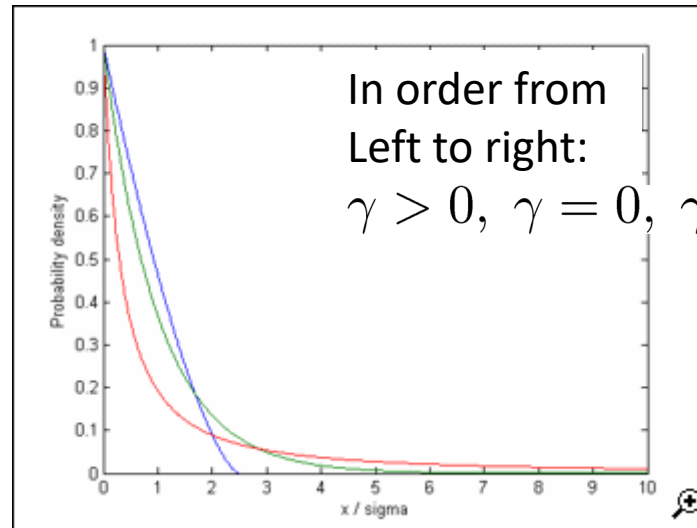


○ Maximum quarterly loss
 ■ excess of the level $u = 1.92$

How large is the risk of a big quarterly loss? **BM**

How large is the risk of a big loss tomorrow? **PoT**

The GP distribution: $H(x) = 1 - \left(1 + \frac{\gamma}{\sigma}x\right)_+^{-1/\gamma}$



density function of Generalized Pareto distribution

$$h(x) = \frac{d}{dx}H(x) = \frac{1}{\sigma} \left(1 + \frac{\gamma}{\sigma}x\right)_+^{-1/\gamma-1} \quad (= \frac{1}{\sigma}e^{-x/\sigma} \text{ if } \gamma = 0)$$

$\gamma \geq 0$ the distribution has left endpoint 0 and right endpoint ∞

$\gamma < 0$ the distribution has left endpoint 0 and right endpoint $\sigma/|\gamma|$

the distribution is “heavytailed” for $\gamma > 0$. Then moments of order greater than $1/\gamma$ are infinite/don't exist, exactly as for the Generalized Extreme Value distribution

- Peaks over thresholds (PoT) method (*Coles p. 74-91*)
- Choose (high) threshold u and from i.i.d observations $Y_1, \dots, Y_n \sim F$ obtain N threshold excesses $X_1 = Y_{t_1} - u, \dots, X_N = Y_{t_N} - u$, where t_1, \dots, t_N are the times of threshold exceedance
- Assume X_1, \dots, X_N are i.i.d and GP distributed and that t_1, \dots, t_N are the occurrence times of an independent Poisson process, so that N has a Poisson distribution
- Use X_1, \dots, X_n to estimate the GP parameters and N to estimate the mean of the Poisson distribution
- Estimate tail $\bar{F}(x) = 1 - F(x) = \bar{F}(u)\bar{F}_u(x - u)$, where $\bar{F}_u(x - u) = 1 - F_u(x - u)$ is the conditional distribution function of threshold excesses, by

$$\hat{\bar{F}}(x) = \frac{N}{n} \hat{\bar{F}}_u(x - u)$$

Details:

Assume the random variable X has d.f. F and let u be a (high) level. The distribution of exceedances then is the conditional distribution of $X - u$ given that X is larger than u , *i.e.* it has d.f.

$$F_u(x) = P(X - u \leq x | X > u) = \frac{P(X - u \leq x \text{ and } X > u)}{P(X > u)} = \frac{F(x+u) - F(u)}{1 - F(u)}$$

$$\text{(and hence } \bar{F}_u(x) = 1 - F_u(x) = \frac{\bar{F}(x+u)}{\bar{F}(u)}).$$

Exercise: Show that if $F(x)$ is a GP distribution, then also $F_u(x)$ is a GP distribution, and express the parameters of $F_u(x)$ in terms of the parameters of $F(x)$ (Treat $\gamma \neq 0$ and $\gamma = 0$ separately.)

More details

N = the (random) number of exceedances of the threshold u by Y_1, \dots, Y_n . The ratio N/n is a natural estimator of $\bar{F}(u)$. Assume we have computed estimators $\hat{\sigma}, \hat{\gamma}$ of the parameters in the GP distribution $\bar{F}_u(x) = H(x)$. Since $\bar{F}(x) = \bar{F}(u)\bar{F}_u(x-u)$, a natural estimator of the “tail function” $\bar{F}(x)$, for $x > u$, then is

$$\hat{\bar{F}}(x) = \frac{N}{n} \hat{H}(x-u) = \frac{N}{n} \left(1 + \frac{\hat{\gamma}}{\hat{\sigma}}(x-u) \right)^{-1/\hat{\gamma}}$$

Solving $\hat{\bar{F}}(x_p) = p$ for x_p we get an estimator of the $1-p$ -th quantile of X :

$$x_p = u + \frac{\hat{\sigma}}{\hat{\gamma}} \left(\left(\frac{n}{N} p \right)^{-\hat{\gamma}} - 1 \right)$$

(Why all this trouble? Why not just estimate $\bar{F}(x)$ by $N(x)/n$? Because if x is a very high level then $N(x)$ is very small or zero, and then this estimator is useless -- and it is such very large x -es we are interested in.)

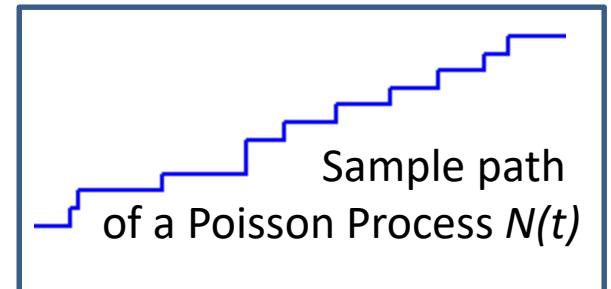
The Poisson process

Model for times of occurrence of events which occur “randomly” in time, with a constant “intensity”, e.g, radioactive decay, times when calls arrive to a telephone exchange, times when traffic accidents occur ... Can be mathematically described as a counting process $N(t) = \# \text{events in } [0, t]$. The counting process $N(t)$ is a Poisson process if

- The numbers of events which occur in disjoint time intervals are mutually independent
- $N(s+t) - N(s)$ has a Poisson distribution for any $s, t \geq 0$, i.e.

$$P(N(s+t) - N(s) = k) = \frac{(\lambda t)^k}{k!} e^{-\lambda t}, \text{ for any } s, t \geq 0, k = 1, 2, \dots$$

λ is the “intensity” parameter. It is the expected number of events in any time interval of length 1.



A connection between the PoT and Block Maxima methods

Suppose the PoT model holds, so values larger than u occur as a Poisson process with intensity λ ; this process is independent of the sizes of the excesses; these are i.i.d. and have a GP distribution

$H(x) = 1 - \left(1 + \frac{\gamma}{\sigma}x\right)_+^{-1/\gamma}$. $M_T =$ the maximum in the time interval $[1, T]$. Then

$$\begin{aligned} P(M_T \leq u + x) &= \sum_{k=0}^{\infty} P(M_T \leq u + x, \text{ there are } k \text{ exceedances in } [0, T]) \\ &= \sum_{k=0}^{\infty} H(x)^k \frac{(\lambda T)^k}{k!} \exp\{-\lambda T\} \\ &= \sum_{k=0}^{\infty} \left(1 - \left(1 + \frac{\gamma}{\sigma}x\right)_+^{-1/\gamma}\right)^k \frac{(\lambda T)^k}{k!} \exp\{-\lambda T\} \\ &= \exp\left\{\left(1 - \left(1 + \frac{\gamma}{\sigma}x\right)_+^{-1/\gamma}\right)\lambda T\right\} \exp\{-\lambda T\} \\ &= \exp\left\{-\left(1 + \frac{\gamma}{\sigma}x\right)_+^{-1/\gamma}\lambda T\right\} \\ &= \exp\left\{-\left(1 + \gamma \frac{x - ((\lambda T)^\gamma - 1)\sigma/\gamma}{\sigma(\lambda T)^\gamma}\right)_+^{-1/\gamma}\right\} \end{aligned}$$