1.) 27 clusters in 6.5 years

likelihood $L(\lambda) = \frac{(\lambda \times 6.5)^{27}}{27!} e^{-\lambda \times 6.5}$

$l(\lambda) = \log L(\lambda) = 27 \log \lambda - \lambda \times 6.5 +$
$+ \log 6.5^{27}/27!$

$\frac{d}{d\lambda} l(\lambda) = \frac{27}{\lambda} - 6.5 = 0 \Rightarrow \hat{\lambda} = \frac{27}{6.5} = 4.154$

$\frac{d^2}{d\lambda^2} l(\lambda) = -\frac{27}{\lambda^2}$ $\quad I(\hat{\lambda}) = -\frac{27}{\hat{\lambda}^2}$

$\hat{SD}(\hat{\lambda}) \approx \sqrt{-\frac{1}{I(\hat{\lambda})}} = \sqrt{\frac{4.154^2}{27}} = 0.639$

confidence interval

$(\hat{\lambda} - 1.96 \times SD(\hat{\lambda}), \hat{\lambda} + 1.96\, SD(\hat{\lambda})) = (2.59, 5.72)$

2.) L_1 = yearly loss line 1

L_2 = - " - line 2

P(maximum of yearly loss for line 1 and yearly loss for line 2 ≤ 160)

$= P(\max\{L_1, L_2\} \leq 160) = P(L_1 \leq 160)P(L_2 \leq 160)$

$$P(L_1 \leq 160) = \exp\left\{-e^{-\frac{160-105}{34}}\right\} = 0.820$$

$$P(L_2 \leq 160) = \exp\left\{-e^{-\frac{160-93}{41}}\right\} = 0.822$$

Hence the probability that the maximum loss is smaller than 160MSEK is $0.820 \times 0.822 = 0.674$

3.) $F_1(x)$ = d.f. of 1-day loss L_1
$F_n(x)$ = d.f. of n-day maximum

$$F_n(x) = F_1(x)^{n\theta} \quad \text{so hence}$$

$$\alpha = F_1(x) \iff F_n(x) = \alpha^{n\theta}$$

$$\hat{F}_n(x) = \begin{cases} \exp\{-e^{-\frac{x-\hat{\mu}}{\hat{\sigma}}}\} & \hat{\gamma} = 0 \\ \exp\{-(1+\frac{\hat{\gamma}}{\hat{\sigma}}(x-\hat{\mu}))^{-\frac{1}{\hat{\gamma}}}\} & \hat{\gamma} \neq 0 \end{cases}$$

- So if $\hat{\gamma} \neq 0$ then

$$\exp\{-(1+\frac{\hat{\gamma}}{\hat{\sigma}}(x-\hat{\mu}))^{-1/\hat{\gamma}}\} = \alpha^{n\hat{\theta}}$$

solving for x gives that

$$VaR_\alpha(L_1) = \frac{\hat{\sigma}}{\hat{\gamma}}\{(-n\hat{\theta}\log\alpha)^{-\hat{\gamma}} - 1\} + \hat{\mu}$$

- If instead $\hat{\gamma} = 0$ then

$$\exp\{-e^{-\frac{x-\hat{\mu}}{\hat{\sigma}}}\} = \alpha^{n\hat{\theta}}$$

solving for x gives

$$VaR_\alpha(L_1) = \cancel{\log(\log\alpha)n\hat{\theta}) + \hat{\mu}}$$
$$= \hat{\sigma}(-\log(-n\hat{\theta}\log\alpha)) + \hat{\mu}$$

(Though one never gets $\hat{\gamma}$ to be exactly 0)

4.) a) L_1 = daily loss

$$P(L_1 > 0.055) = P(L_1 > 0.055 \mid L_1 > 0.04)\,P(L_1 > 0.04)$$

$$= P(L_1 - 0.04 > 0.015 \mid L_1 > 0.04)\,P(L_1 > 0.04)$$

Inserting estimates one obtains that the probability that a daily loss is larger than 0.055 is estimated by

$$\left(1 + \cancel{\frac{0.024}{0.51}} \times 0.015\right)^{-\frac{1}{0.51}} \times 0.035 = 0.0203$$

$$\frac{0.51}{0.024}$$

4.) b)

By part a) one has to solve the equation

$$P(L_1-0.04 > X-0.04 \mid L_1>0.04)\,P(L_1>0.04)=1-\alpha$$

i.e. the equation

$$\left(1+\frac{0.51}{0.024}\times(X-0.04)\right)^{-\frac{1}{0.51}}\times 0.035 = 0.01$$

This gives

$$\mathrm{VaR}_{0.99}(L_1) = 0.082$$

and hence the 99% VaR for the portfolio is

$$\text{SEK } 1.5\times10^6\times0.082 = \text{SEK } 0.123\times10^6$$

5.)

$$P(\text{no defaults}) = P(\text{no defaults} \mid Z=1)\,P(Z=1) + P(\text{no defaults} \mid Z=2)\,P(Z=2)$$

$$= (1-0.03)^{25} \times 0.8 + (1-0.09)^{25} \times 0.2$$

$$= 0.392$$

6.) $N = \#$ defaults
L = loss

$$L = 0.6 \times 10^6 \times N \quad (*)$$

LPA: $P\left(\frac{N}{1000} \le x\right) \approx P(p(Z) \le x)$
$= P(Z \le p^{-1}(x))$

logit-normal model: solving

$$y = p(Z) = \frac{1}{1+\exp(-(\mu+\sigma Z))}$$

gives that

$$p^{-1}(x) = \frac{1}{\sigma}\left(\log\frac{x}{1-x} - \mu\right)$$

$$P(L \le 80 \times 10^6) \underset{(*)}{=} P\Big(\frac{N}{1000} \le \underbrace{\frac{80\times 10^6}{1000 \times 0.6 \times 10^6}}_{=0.133}\Big)$$

$$= N\Big(\underbrace{\frac{1}{1.09}\left(\log\frac{0.133}{1-0.133} + 2.537\right)}_{=0.607}\Big)$$

$= 0.728$

$P(30\times 10^6 \le L \le 80\times 10^6) = P(L \le 80 \times 10^6)$
$- P(L \le 30 \times 10^6) = 0.728 - 0.356$

computed in same way

$= 0.372$

7.) a.) $N = \#$ defaults

$L = \text{loss} = 0.6 \times N \quad (M\$)$

LPA as in 6.):

$$\mathbb{P}(L \le x) = \mathbb{P}\Big(\frac{N}{1000} \le \underbrace{\frac{x}{600}}_{=y}\Big)$$

$$\approx \mathbb{P}(p(Z) \le y)$$

In Merton framework

$$\mathbb{P}(p(Z) \le y) = \mathbb{P}\Big(N\Big(\frac{N^{-1}(\bar{p}) - \sqrt{\varrho}\, Z}{\sqrt{1-\varrho}}\Big) \le y\Big)$$

$$= \mathbb{P}\Big(-Z \le \frac{\sqrt{1-\varrho}\, N^{-1}(y) - N^{-1}(\bar{p})}{\sqrt{\varrho}}\Big)$$

= normal d.f is symmetric

$$= N\Big(\frac{\sqrt{1-\varrho}\, N^{-1}(y) - N^{-1}(\bar{p})}{\sqrt{\varrho}}\Big) = 0.99$$

$$\Longleftrightarrow$$

$$\frac{\sqrt{1-0.14}\, N^{-1}(y) - N^{-1}(0.04)}{\sqrt{0.14}} = 2.326$$

Solving for y given that

$$y = N\left(\underbrace{\frac{\sqrt{0.14}\times 2.326 + \underbrace{N^{-1}(0.04)}_{=-1.751}}{\sqrt{0.86}}}_{-0.950}\right)$$

$= 0.171$

$$VaR_{0.99}(L) = 600 \times 0.171 = 0.102 \ (M\$)$$

7.) b.) From a) we have that

$$F_M(x) = P(P(Z) \le x) =$$
$$= N\left(\frac{\sqrt{1-\rho}\, N^{-1}(x) - N^{-1}(\bar{p})}{\sqrt{\rho}}\right)$$
$$= N\left(\frac{0.927\, N^{-1}(x) + 1.751}{0.374}\right)$$

and computations such as in 6.) give that

$$F_{\log N}(x) = N\left(\frac{1}{\sigma}\left(\log\frac{x}{1-x} - \mu\right)\right)$$

Thus setting $F_M(x_i) = F_{\log N}(x_i)$ for $x_i = 0.1$ and $x_i = 0.9$ gives the equations

$$\begin{cases} \dfrac{0.927 \times (-1.281) + 1.751}{0.374} = \dfrac{1}{\sigma}(-2.197 - \mu) \\[2ex] \dfrac{0.927 \times 1.281 + 1.751}{0.374} = \dfrac{1}{\sigma}(2.197 - \mu) \end{cases}$$

which have the solutions

$$\begin{cases} \mu = -3.240 \\ \sigma = 0.692 \end{cases}$$

$$F_{\log N}(x) = N\left(\frac{1}{\sigma}\left(\log\frac{x}{1-x} - \mu\right)\right) = 0.99$$

$$\Longleftrightarrow$$

$$2.326 = \frac{1}{0.692}\left(\log\frac{x}{1-x} + 3.240\right)$$

$$\log\frac{x}{1-x} = -1.630$$

$$x = 0.164$$

$$VaR_{\log L,\,0.99}(L) = 600 \times 0.164 = 98.4 \text{ MSEK}$$