Exam: Finansiell Risk, MVE 220/MSA400
Monday, June 3, 14.00-18.00
Jour: Barbara Schnitzer ankn 5325

Allowed material: List of Formulas, Normal Table, Chalmers allowed calculator.

Good Luck!

1 In an analysis of OMX daily negative returns during a six and a half year period, a threshold was selected and a standard PoT analysis with declustering was made. During this period 27 clusters of exceedances and 63 exceedances were found. The occurrence of clusters was modeled by a Poisson process with intensity λ/year. Write down the likelihood function for this Poisson process, find the maximum likelihood estimator of λ, and use observed information to construct a 95% confidence interval for λ. (2p)

2 An insurance company is interested in the joint risk connected with two lines of bussiness. From experience it is known that the maximum yearly loss in MSEK (million Swedish crowns) for Line 1 has a GEV distribution with parameters $\mu_1 = 105, \sigma_1 = 34, \gamma_1 = 0$ and that the maximum yearly loss in MSEK for Line 2 has a GEV distribution with parameters $\mu_2 = 93, \sigma_2 = 41, \gamma_2 = 0$, and that the losses are independent of one another. Find the probability that the yearly losses from Line 1 and Line 2 both are smaller than 160 MSEK (i.e., equivalently phrased, find the probability that the maximum of the yearly loss from Line 1 and from Line 2 is smaller than 160 MSEK.) (2p)

3 Suppose a GEV distribution has been fitted to the n-day maxima of the returns of a stock index, and that the estimated parameters of the GEV distribution are $\hat{\mu}, \hat{\sigma}$ and $\hat{\gamma}$. Further assume that the estimated value of the extremal index is $\hat{\theta}$. Find a formula for VaR_α for the 1-day returns. (2p)

4 Suppose daily losses (= -returns) of a stock are independent and identically distributed and that the excesses of the level 0.04 follow a GP distribution. Further assume that in five years of data 3.5% of the observed losses had been larger than 0.04 and that the estimated scale and shape parameters of the GP distribution were $\hat{\sigma} = 0.024$ and $\hat{\gamma} = 0.51$.

a) Find an estimate of the probability that a daily loss is larger than 0.055 (2p)

b) Suppose an investor has bought 1.5 million crowns worth of this stock. Find an estimate of the 99% daily VaR for this investment. (2p)

5 Consider a static credit portfolio with $m = 25$ obligors. We model this portfolio as a mixed binomial model and let Z be the random variable representing the background variable affecting all obligors in the portfolio. Let $X_i = 1$ if obligor i defaults and $X_i = 0$ otherwise. Furthermore, let $p(Z) = \mathbb{P}[X_i = 1 \mid Z]$. In this model Z has two states $\{1, 2\}$ where $\mathbb{P}[Z = j]$ and $p(j)$ for $j = 1, 2$ are given by Table 1. Compute the exact probability of having no defaults in this portfolio within one year.

state j	$j = 1$	$j = 2$
$\mathbb{P}[Z = j]$	0.8	0.2
$p(j)$	0.03	0.09

Table 1: The values $\mathbb{P}[Z = j]$ and $p(j)$ for $j = 1$ and $j = 2$.

(3p)

6 Consider a static credit portfolio with $m = 1000$ obligors which we model as a mixed binomial model with a logit-normal mixing distribution with parameters $\mu = -2.537$ and $\sigma = 1.09$. Each loan has notional 1 million SEK and the individual loss is $\ell = 60\%$. Use the LPA-approximation formula to compute the probability that within one year, the total portfolio credit loss will be more than 30 million SEK but less than 80 million SEK. (3p)

7 A credit portfolio manager wants to compare the 1-year $\text{VaR}_\alpha(L)$ for two different static credit portfolio models: the mixed binomial logit-normal model versus the mixed binomial model inspired by the Merton framework. The manager has estimated the one-year parameters $\bar{p}$ and ρ in the Merton framework to $\bar{p} = 4\%$ and $\rho = 14\%$ which gives him a certain VaR_α-value in the Merton setup for $\alpha = 99\%$. The manager now wants to compute the corresponding VaR in a mixed binomial logit-normal model under similar conditions as in the Merton framework and decides to calibrate the logit-model against the mixed binomial Merton model so that

$$F_M(x_i) = F_{\text{logN}}(x_i) \quad \text{for } i = 1, 2 \text{ where } x_1 = 0.1 \text{ and } x_2 = 0.9$$

where $F_M(x)$ and $F_{\text{logN}}(x)$ are the LPA-distributions for the fractional number of defaults in the credit portfolio, for a mixed binomial Merton model and for the mixed binomial logit-normal model, respectively. The credit portfolio has 1000 obligors, each loan has notional 1 million US-dollars and the individual loss is $\ell = 60\%$.

a) Use the LPA to compute the $\mathrm{VaR}_{99\%}$ in the Merton-inspired model (1p)

b) If the manager uses the LPA-approach what will the 1-year $\mathrm{VaR}_{99\%}(L)$ be in the mixed binomial logit-normal model which was calibrated to the Merton framework as above? (3p)