Computer project 2

To study the performance of MCMC we propose the following toy example.

1. Let (X_1, X_2) have bivariate normal distribution with mean 0 and covariance matrix
$$\begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}.$$
Simulate (X_1, X_2) for some values of ρ of your own choice, using the Gibbs sampler described on page 31-32 in the lecture notes from Mats Viberg's lecture (available on the homepage).

   When do you think that the chain has converged, i.e. when should you start to sample?

2. Use your sample to calculate the means, the variances and the correlation. You can also check that the marginal distributions are normal by using a probability plot or a quantile plot.

3. Use your sample to investigate the distribution of the sum $Y_1 = X_1 + X_2$ and the difference $Y_2 = X_1 - X_2$ of the components X_1 and X_2. Estimate the variances of Y_1 and Y_2 and the correlation of Y_1 and Y_2.

4. Theoretical question. Suppose that the Markov chain in the Metropolis-Hastings algorithm on page 22 (Mats' lecture notes) is irreducible and non-null persistent. Show that the detailed balance equations (6.5.3) in Grimmett & Stirzaker are satisfied. Hence by Theorem 6.5.4, π is a stationary distribution for the chain. And Theorem 6.4.3. tells us that it is unique.