MVE251 - Probability and random processes, advanced level

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Signals and Systems

Why Stochastic Processes?

General Motivation

- Modeling measured signals (and many other phenomena) as random entities is **successful**!
- Important to understand the underlying mechanism and formalism – we are not dealing with the **true** nature of signals!
- Many pitfalls, mistakes are common!
- Ph.D. students in the ”EE area” need at least one mathematically oriented course!

Application areas

- Understand how signals and systems behave:
  - Measured data and relations between signals
  - Continuous and discrete phenomena
  - Stationary and non-stationary
- Derive and understand methods for estimation, detection and classification
  - Parametric modeling from data
  - Non-parametric descriptions; Fourier, Wavelets etc.
  - Digital Communication receivers
  - Fault detection and event classification
  - Etc...

Examples

- ARMA-Modeling: $A(z)y(n) = B(z)e(n)$
  - Coefficients a_k, b_k estimated using sample based auto-covariance function estimates
  - What are the statistical properties of the estimates?
  - How reliable is the spectrum estimate?
- Detection: received signal *x(t)* is a distorted (channel and noise) version of an encoded bit stream
  - What is the best way to determine the bits?
  - What is the probability of error?
  - What if channel unknown, non-stationary etc.?

- Computer-intensive statistical methods
  - Bayesian estimation/detection: high- dimensional integrals over complicated PDFs
  - Monte-Carlo methods (applications in simulation and detection/estimation)
  - Particle filters for tracking applications
  - Importance/Rejection sampling
  - Markov-Chain Monte Carlo (MCMC) methods: Metropolis-Hastings alg., Gibbs sampling etc.
  - Cross-validation, jack-knifing, bootstrap etc.

Increasing importance due to faster computers!

What do engineers need to know?

- The basics: What is probability? Foundations?
- Gut feeling for properties of random signals (correlation, spectrum etc.)
- Calculation of probabilities, e.g. performance of a communication receiver for a random channel
- Understand concepts – for example: how to treat large random samples (convergence)?
- Master statistical tools – necessary to understand properties of your algorithms!
- Know when you cheat (or when it is time to ask the experts)