# EXAMPLE OF WRITTEN EXAMINATION

**1** The annual maximum sea-level, $X$ (in meters), at Port Pirie, located in the southern part of Australia, has been recorded for a period of years. Gumbel distribution with the parameters $a$ and $b$ is used to model $X$. Based on data year 1923-1987, the ML-estimates are $a^* = 0.198$, $b^* = 3.87$.

**(a)** Estimate the hundred year sea-level, $x_{100}$, at Port Pirie. (5 p)

**(b)** Give an approximate 99% confidence interval for $x_{100}$. (5 p)

**2** A new heating system has been installed. It is a modern system which can give a warning when reliability/efficiency is decreasing and then a major service is recommended. Suppose that service times form a stationary Poisson stream of events with an unknown intensity $\lambda$ [year$^{-1}$]. Depending on quality of fuel and amount of needed energy, the intensity $\lambda$ may vary. The dealer claims that on average, service is needed once in two years, i.e. $\lambda = 0.5$ year$^{-1}$.

**(a)** When ordering the system there is an option for a constant price of service, $c = 4000$ SEK. Since $\lambda$ is intensity (frequency) of services needed, the average cost per year is $c\lambda$. What is the predicted yearly cost based on the information given by the dealer? (3 p)

**(b)** Suppose that you choose the option of constant service price and that service was needed once in the first 6 months of use. How does this information affect your predicted yearly service cost? (Hint: Use a suitable exponential prior, update and then compute $\mathsf{E}[c\Lambda]$.) (7 p).

**3** In the field of life insurance standardized death-rates are often utilized. One example is the N-1963 standard according to which the death rate or "failure-intensity" for females is equal to

$$\lambda_{\mathrm{F}}(t) = \alpha + \beta \mathrm{e}^{(t-t_0)/c}, \qquad t > 0.$$

It was found the for a certain population the estimated parameters are $\alpha^* = 9 \cdot 10^{-4}$, $\beta^* = 4.4 \cdot 10^{-5}$, $c^* = 10.34$ and $t_0^* = 3$. Estimate the risk for death of 80 years old women in 10 years. (Hint: compute first probability that 80 years old women will live to be 90 years old.) (10 p)

**4** Consider the data of failures of the air-condition system for a fleet of Boeing 720 plane. The time distance between failures (in hours) are

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 50 | 44 | 102 | 72 | 22 | 39 | 3 | 15 | 197 | 188 | 79 | 88 |
| 46 | 5 | 5 | 36 | 22 | 139 | 210 | 97 | 30 | 23 | 13 | 14 |

The hypothesis is that events of failures can be modelled as Poisson stream with constant intensity of failures denoted by $\lambda$. Test the hypothesis about the model (give the name of the test you are using for this purpose). If the test does not rejected the Poisson model estimate the intensity $\lambda$. (20 p)

**5** In a 10-year period there were 57 events of large fires in industrial buildings of a certain type (similar sizes and utilization) in Scania region. In that period there were approximately 260 industial buildings of that type. Suppose that occurences of industry-fires follow Poisson process, with the same intensity for all buildings in that population, while losses in a fire $X$ (SEK) are lognormal distributed, i.e. $\ln X \in \mathrm{N}(m_X, \sigma_X)$, with ML-estimates $m_X^* = 15$ and $\sigma_X^* = 1.32$.

**(a)** Estimate the intensity $\lambda$ of the Poisson process. (5 p)

**(b)** What is the probability that the loss due to one fire in a building exceeds 10 million SEK, i.e. estimate $\mathsf{P}(X_i > 10^7)$? (5 p)

**(c)** Suppose that losses due to different fires are independent and identically distributed. What is the probability that a company that resides in a single industrial building experiences at least one fire with a loss exceeding 10 million SEK within next 10 years from now? (10 p)

**6** Ekofisk is an oil field in the Norwegian sector of the North Sea. Discovered in 1969, it remains one of the most important oil fields in the North Sea. The present time horizon for the operation of Ekofisk is 2028. A challenging problem at Ekofisk has been the subsidence of the seabed. In 1986, deck structures of a number of platforms had to be elevated by 6 meters. When measured against the sea level at that time, it was 27 meters above the sea level. However, the subsidence today is 8.5 meters so the deck structure is now 18.5 meters above sea level. It is assumed that the annual maximum wave height, $H$ (meters above sea level), is Gumbel distributed, with the estimated parameters $a^* = 1.95$ and $b^* = 5.2$.

**(a)** Estimate present yearly probability of wet deck, i.e. of waves reaching the platform deck structures in the next 12 months (assume that for this period the subsidence can be considered constant. (7 p)

**(b)** Estimate the Cornell safety index $\beta_C$ for the risk of waves reaching the platform deck structures in the next 12 months. (3 p)

**(c)** Assume that the subsidence, $S$ (in meters), of the seabed year 2028 can be modeled as the following function of a random variable $X$

$$S = 13.3 + 0.5\sqrt{X},$$

with $\mathsf{E}(X) = 1.5$ and $\mathsf{V}(X) = 0.5$. (The random variable $X$ represents a certain parameter the nature of which is irrelevant for the problem.)
How much one need to increase the deck level to keep the same Cornell safety index for waves reaching the platform deck structures in year 2028?
(10 p)