# Tentamen
# MVE300 Sannolikhet, statistik och risk

### xxxx-xx-xx kl. 830-13.30

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**Hjälpmedel:** Inga hjälpmedel.

Denna tentamen utgoer, tillsammans med godkaent i Matlabmomentet, grunden foer betygssaettning. Foer betyg 3 kraevs minst 20 poaeng, foer betyg 4 minst 30 poaeng och foer betyg 5 minst 40 poaeng.

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**1**. (6p) Two coins are flipped. One of the coins is such that it shows heads with probability 1/3 whereas the second one shows heads with probability 2/3. Given that you get exactly one heads, what is the conditional probability that it was the second coin?

**2**. (6p) Among twelve light bulbs, there are five defective ones. If one picks uniformly at random four of these light bulbs, what is the probability of getting $x$ defected ones, $x = 0, 1, 2, 3, 4$?

**3**. (6p) Suppose that a random number generator uniform numbers among $1, 2, \ldots, 9$. What is the probability that the product of $n$ such independent random numbers, is divisible by 10?

**4**. (7p) Let $X_1, X_2, \ldots$ be independent and identically distributed random variables with expectation $\mu$. Let $N$ be a positive integer valued random variable such that $\mathbb{E}[N] < \infty$ and such that $I_{\{N \geq n\}}$ is independent of $X_n$ for all $n$. Prove that

$$\mathbb{E}[\sum_{i=1}^{N} X_i] = \mu \mathbb{E}[N].$$

**5**. (7p) Assume that the traffic at a certain point at a certain road is a Poisson process with intensity $c$. After 30 minutes, you have observed 214 vehicles pass.

(a) Make a maximum likelihood of the intensity $c$,

(b) Use the central limit theorem to give an approximate 95% confidence interval for $c$

**6**. (6p) In a sample of 15 normally disributed random variables with unknown expectation $\mu$ and variance $\sigma^2$, the sample mean was 10.3 and the sample variance $s^2$ was 0.13.

(a) Make a symmetric confidence interval for $\mu$ at the 99% confidence level.

(b) Test the null hypothesis $\mu = 10$ against the alternative hypothesis $\mu > 10$ at the 5% significance level.

**7**. (6p) Show that the Gumbel distribution is max stable, i.e. if $X$ and $Y$ are two independent Gumbel distributed random variables with the same scale parameter, then $\max(X, Y)$ is Gumbel with the same scale parameter as $X$ and $Y$. Recall that the distribution function in the Gumbel distribution is

$$F(x) = \exp(-e^{-(x-b)/a}),$$

where $a$ is the scale parameter.

**8**. (6p) Let $X_1, \ldots, X_n$ be a sample of some distribution. Show that the sample variance

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \overline{X})^2$$

is an unbiased estimator of $\sigma^2 = \mathbb{V}\mathrm{ar}(X_1)$. Show also that the sample standard deviation $s$ satisfies $\mathbb{E}[s] \leq \sigma$.

Lycka till!
Johan Jonasson