Tentamen
MVE300 Sannolikhet, statistik och risk

xxxx-xx-xx kl. 830-13.30

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Hjälpmedel: Inga hjälpmedel.

Denna tentamen utgoer, tillsammans med godkaent i Matlabmomentet, grunden foer betygssaettning. Foer betyg 3 kraevs minst 20 poaeng, foer betyg 4 minst 30 poaeng och foer betyg 5 minst 40 poaeng.

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1. (6p) Two coins are flipped. One of the coins is such that it shows heads with probability 1/3 whereas the second one shows heads with probability 2/3. Given that you get exactly one heads, what is the conditional probability that it was the second coin?

2. (6p) Among twelve light bulbs, there are five defective ones. If one picks uniformly at random four of these light bulbs, what is the probability of getting x defected ones, $x = 0, 1, 2, 3, 4$?

3. (6p) Suppose that a random number generator uniform numbers among $1, 2, \ldots, 9$. What is the probability that the product of n such independent random numbers, is divisible by 10?

4. (7p) Let $X_1, X_2, \ldots$ be independent and identically distributed random variables with expectation μ. Let N be a positive integer valued random variable such that $\mathbb{E}[N] < \infty$ and such that $I_{\{N \geq n\}}$ is independent of X_n for all n. Prove that

$$\mathbb{E}[\sum_{i=1}^{N} X_i] = \mu \mathbb{E}[N].$$

5. (7p) Assume that the traffic at a certain point at a certain road is a Poisson process with intensity c. After 30 minutes, you have observed 214 vehicles pass.

(a) Make a maximum likelihood of the intensity c,

(b) Use the central limit theorem to give an approximate 95% confidence interval for c

6. (6p) In a sample of 15 normally distributed random variables with unknown expectation μ and variance σ^2, the sample mean was 10.3 and the sample variance s^2 was 0.13.

(a) Make a symmetric confidence interval for μ at the 99% confidence level.

(b) Test the null hypothesis $\mu = 10$ against the alternative hypothesis $\mu > 10$ at the 5% significance level.

7. (6p, only TM) Show that the Gumbel distribution is max stable, i.e. if X and Y are two independent Gumbel distributed random variables with the same scale parameter, then $\max(X, Y)$ is Gumbel with the same scale parameter as X and Y. Recall that the distribution function in the Gumbel distribution is

$$F(x) = \exp(-e^{-(x-b)/a}),$$

where a is the scale parameter.

8. (6p) Let $X_1, \ldots, X_n$ be a sample of some distribution. Show that the sample variance

$$s^2 = \frac{1}{n-1} \sum_{i=1}^{n} (X_i - \overline{X})^2$$

is an unbiased estimator of $\sigma^2 = \mathbb{V}\mathrm{ar}(X_1)$. Show also that the sample standard deviation s satisfies $\mathbb{E}[s] \leq \sigma$.

Lycka till!
Johan Jonasson