Computer exercise 3
Covariance estimation and kriging
Statistical image analysis, TMS016

1 Introduction

The purpose of this computer exercise is to give an introduction to parameter estimation and kriging for Gaussian random field models for spatial data.

Before you begin, download the Matlab files for the exercise from the course homepage. When in doubt about how to use a specific function, use `help` and `doc` to get more information.

2 Data generation

Use the methods from computer exercise 2 to simulate a Gaussian random field x on a regular $n \times n$ lattice (let n be at least 50). Use a Matérn covariance function and assume a regression for the mean using the two basis functions $B_1(s) = 1$ and $B_2(s) = x$ (here x is the x-coordinate of $s = (x, y)$). Based on this simulation, construct N observations (let N be around 500)

$$Y_i = X(s_i) + \sigma_i, \quad i = 1, \ldots, N$$

where $\sigma_i \sim \mathsf{N}(0, \sigma_e^2)$ are independent measurement noise terms. Choose the observation locations $s_1, \ldots, s_N$ by randomly selecting N of the locations on the lattice. This can be done by

```
>> ind = randperm(n*n);
>> ind_o = ind(1:n_obs);
>> loc_o = loc(ind_o,:);
```

Plot the simulated field using `imagesc` as well as the observations using `scatter`. Using the `hold on` command, you can plot the two in the same figure. You might then have to set the two arguments `x` and `y` in `imagesc` so that the two plots use the same scale on the x and y axes.

3 Parameter estimation

We will now use the simulated data `y` to estimate the model parameters using the classical geostatistical approach.

- Start by estimating the mean μ using least-squares, and compute the residuals `e = y - mu`. Compare the estimated regression parameters to the true ones.
- Use the function `emp_variogram` to compute a binned estimate of the variogram and compare with the true variogram that can be computed using the function `matern_variogram`.
- Use the function `cov_ls_est` to perform least-squares estimation of a Matérn variogram to the binned estimate. Plot it together with the true variogram and the binned estimate.
- Update the estimate of the regression parameters using GLS. Compare with the OLS estimate as well as the true parameters.

4 Kriging prediction

We will now use the estimated model parameters to perform kriging prediction. We reconstruct the field at all locations on the grid.

- Compute the needed matrices $\mathbf{\Sigma}_o$, $\mathbf{\Sigma}_p$, $\mathbf{\Sigma}_{op}$, $\mathbf{B}_o$, and $\mathbf{B}_p$. Note that you allready have computed some of these before. Also note that there is no nugget in $\mathbf{\Sigma}_p$ since we want to predict the latent field.
- Compute the kriging predictor $\hat{X}$ and compare with the true field.

5 Likelihood-based parameter estimation

- Redo the estimation of the parameters using maximum-likelihood. This can be done using the `cov_ml_est` function. Compare the results to those obtained using least-squares.
- Re-compute the kriging predictor based on the ML parameter estimates and compare with the previous predictor. Is there a large difference?
- If time permits, also compute the variances of the kriging predictions based on the two parameter estimates and compare the differences.