Computer exercise 8
Parameter estimation for discrete Markov random fields
Statistical image analysis, TMS016

1 Introduction

The purpose of this computer exercise is to give an introduction to parameter estimation for discrete Markov random field models. This will be used in the second part of the project assignment.

Before you begin, download the Matlab files for the exercise from the course homepage. When in doubt about how to use a specific function in Matlab, use `help` and `doc` to get more information.

2 Estimation on simulated data

- Use `mrf_sim` to simulate a discrete Markov random field. For example:

```
K = 3;                        %number of classes
sz = [100 100];               %size of the image

N = [0 1 0;1 0 1;0 1 0];      %Neighborhood structure
alpha = log(ones(1,K)/K);     %alpha parameters
beta = 2*eye(K);              %beta parameters

%set a starting value
x = randi(K,[sz(1) sz(2)]);
z0 = zeros(sz(1),sz(2),K);
for i=1:sz(1)
  for j=1:sz(2)
   z0(i,j,x(i,j)) = 1;
  end
end
%simulate z using Gibbs sampling
[z, Mz,ll] = mrf_sim(z0,N,alpha,beta,100,2);
```

- Now for each class, simulate some 3-dimensional Gaussian data and combine into an image

```
yi = cell(1,K);
yi{1} = mvnrnd([1;0;0],0.2*eye(3),sz(1)*sz(2));
yi{2} = mvnrnd([0;1;0],0.2*eye(3),sz(1)*sz(2));
yi{3} = mvnrnd([0;0;1],0.2*eye(3),sz(1)*sz(2));
y = zeros([sz 3]);
for k=1:K
  y = y + bsxfun(@times,z(:,:,k),reshape(yi{k},[sz 3]));
end
```

- Plot `z` and the simulated data, and then estimate the parameters of the model based on this data. The estimation can be done using the function `mrf_sgd`:

```
opts = struct('plot',2,'N',N,'common_beta',1);
[theta,alpha,beta,cl,p]=mrf_sgd(y,K,opts);
```

See the help file for the function to see the possible options that can be supplied using the `opts` structure.

- Besides the parameter estimates, the function returns the maximum aposteriori classification of the image, `cl`, and the class probabilities, `p`. Plot the MAP classification and compare to the original MRF.
- Try changing the parameters for the simulated data and see how well you can estimate the parameters and recover the original segmentation of the image.
- Do the second part of the project assignment, where you will try the estimation method above for real data.