Statistical Image Analysis

Lecture 12: Point processes

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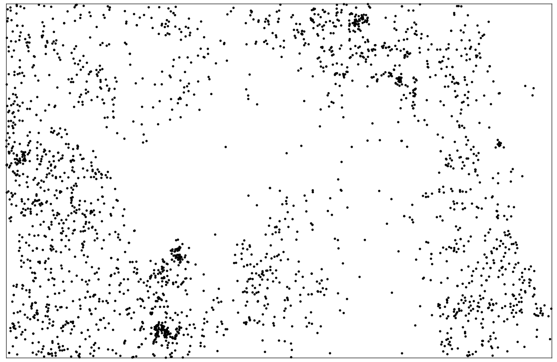
Example: Data



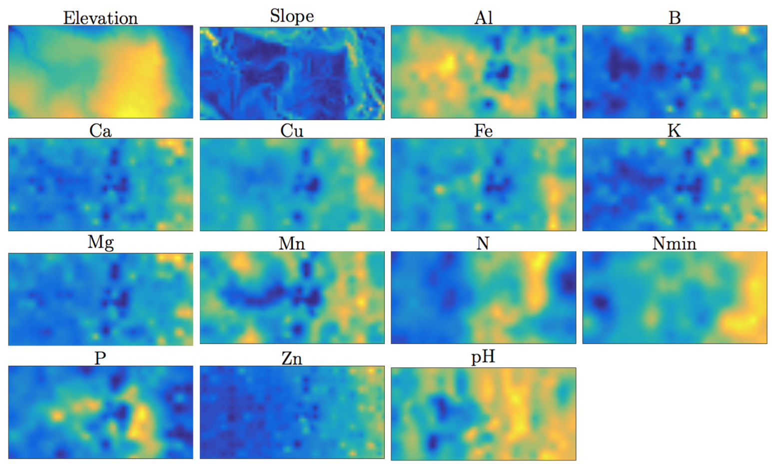
The locations of the tree species Beilschmiedia Pendula in the tropical rainforest plot on Barro Colorado Island.

Types of spatial data

Three main types of data in spatial statistics

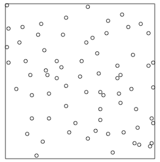
- Continuously indexed data.
  - We have observations at some fixed locations $\mathbf{s}_1, \ldots, \mathbf{s}_N$ of a random field $X(\mathbf{s})$ where $\mathbf{s}$ is in some region $D \subseteq \mathbb{R}^2$. X is a random field defined on D.
- Discretely indexed data.
  - We have observations at some fixed locations $\mathbf{s}_1, \ldots, \mathbf{s}_N$ of a random field $X(\mathbf{s})$ where $\mathbf{s}$ is in some discrete set of locations $\tilde{D}$, such as a regular lattice. X is a random field defined on $\tilde{D}$.
- Point process data.
  - We have observations $\mathbf{s}_1, \ldots, \mathbf{s}_N$ indicating where something occurred. We want to draw conclusions about the process based on these locations, which are now considered to be random.

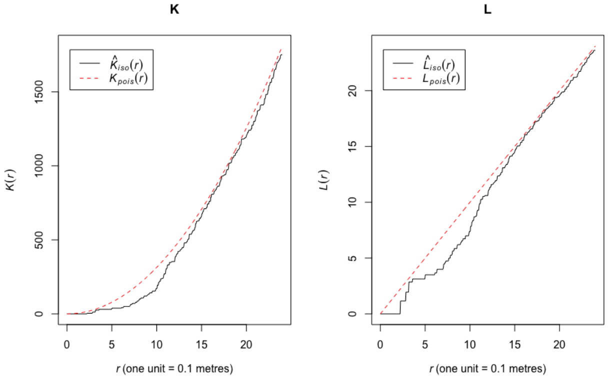
Example: Covariates



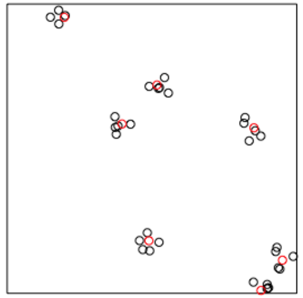
Possible covariates that can be used for drawing conclusions on the association of habitat preferences.

Example: Swedish pine saplings





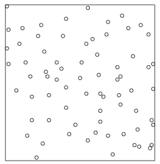
Neyman-Scott process

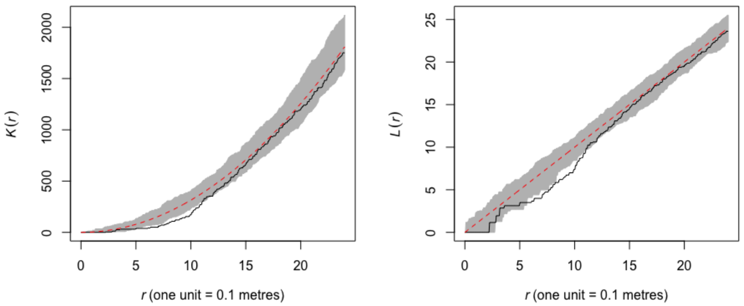


Hierarchical construction

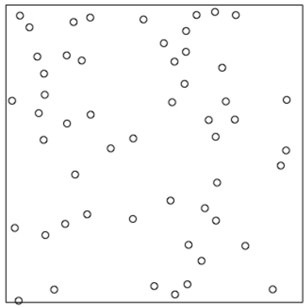
- Simulate a "mother process" Z as a Poisson process.
- For each point $z_i \in Z$, simulate n_i independent "daughter points" x_j from a distribution $\pi(x|z_i)$ and remove Z.
- For example: Let $n_i = 5$ and take $x|z_i$ be uniform in a disc of radius 0.05 centered in z_i.
- Extension: take n_i from some distribution $\pi(n)$.

Example: Swedish pine saplings



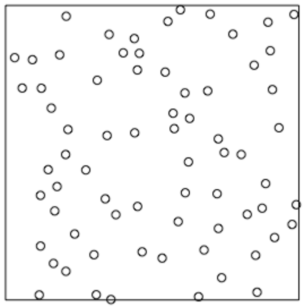


Matérn Type 1 inhibition process



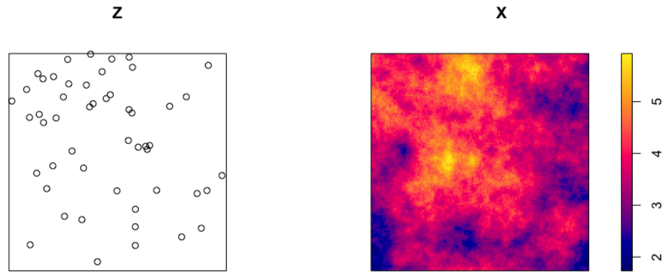
- Simulate a homogeneous Poisson process Z.
- Delete any point in Z that lies closer than a distance r from the nearest other point.
- Thus, pairs of close neighbours annihilate each other.

Matérn Type 2 inhibition process



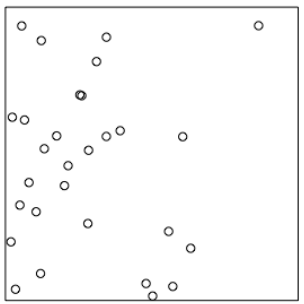
- Simulate a homogeneous Poisson process Z.
- Mark each point in Z by "ages", which are independent and uniformly distributed numbers in $[0, 1]$.
- Delete any point in Z that lies closer than a distance r from another point that has has a higher age.

Log-Gaussian Cox process



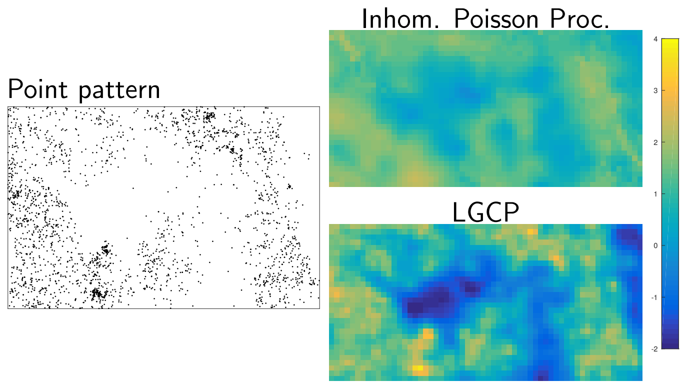
- Hierarchical model, where X is a Gaussian random field and $Z|X$ is an inhomogeneous Poisson process where $\lambda(x, y) = \exp(X(x, y))$.
- Example: X is a Gaussian random field with mean 3 and an exponential covariance function.

Inhomogeneous Poisson process



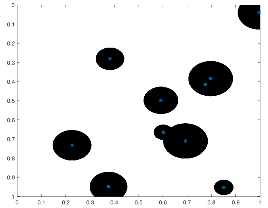
- A poisson process with a spatially varying intensity function.
- Example $\lambda(x, y) = 100 \exp(-3x)$.

Example: Barro Colorado Island



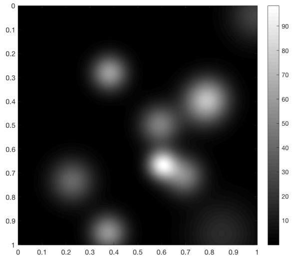
- For the inhomogeneous Poisson process, the intensity function is a regression on the soil covariates.
- For the LGCP the intensity function is the regression plus a mean-zero Gaussian random field.

Marked point processes and boolean models



- For each point z_i in the point pattern, assign a random "mark" M_i.
- Can be used to include more information in spatial point pattern data (such as tree size or age).
- Models like this can also be used to build more complicated spatial models, such as Boolean models:
    - For each z_i, define a disc D_i centered at the point, with a random radius r_i.
    - Define a binary image by letting the pixels have value one in the region $\cup_i D_i$, and zero elsewhere.

Models for grayscale images based on point processes



- We can extend the boolean model to a model for grayscale images by replacing the discs with kernel functions.
- Example: Let the value at location $\mathbf{s}$ in the image be

$$I(\mathbf{s}) = \sum_i \pi_G(\mathbf{s}; z_i, r_i\mathbf{I})$$

where r_i are some positive random variables.