Statistical Image Analysis

Lecture 8: Markov random fields

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Constructing Markov random fields

- How can we define a valid random field model for $\mathbf{z}$?
- Recall that we defined GMRFs using undirected graphs $\mathcal{G} = (E, V)$.
- Typically, we have the set of vertices V as the pixels in an image, and the set of edges E defines the dependence structure.
- We defined GMRFs using local constructions, such as the CAR models where we specified the joint distribution through the conditionals $\pi(x_i|x_{-i}) = \pi(x_i|x_{N_i})$.
- Today we will use local constructions to define discrete valued MRFs.
- Next lecture, we will look at parameter estimation and how to use the models for image segmentation.

Markov random field mixture models

- Hierarchical model for pixel values given classes:

$$\pi(\mathbf{Y}_i|z_i = k) \sim \mathsf{N}(\boldsymbol{\mu}_k, \boldsymbol{\Sigma}_k)$$

$$\pi(z_i) = \begin{cases} \pi_1 & \text{if } z_i = 1 \\ \pi_2 & \text{if } z_i = 2 \\ \vdots & \\ \pi_K & \text{if } z_i = K \end{cases}$$

- Assuming independence between the pixels is not realistic!
- In a Markov random field mixture model, we use the model

$$\pi(\mathbf{Y}_i|z_i = k) \sim \mathsf{N}(\boldsymbol{\mu}_k, \boldsymbol{\Sigma}_k)$$
$$\mathbf{z} \sim \pi(\mathbf{z})$$

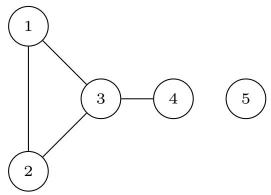
  here $\mathbf{z} = (z_1, \ldots, z_n)$ is a random field that takes values in $\{1, \ldots, K\}$, with density $\pi(\mathbf{z})$.
- Spatial dependencies modeled through $\pi(\mathbf{z})$.

Cliques

Definition

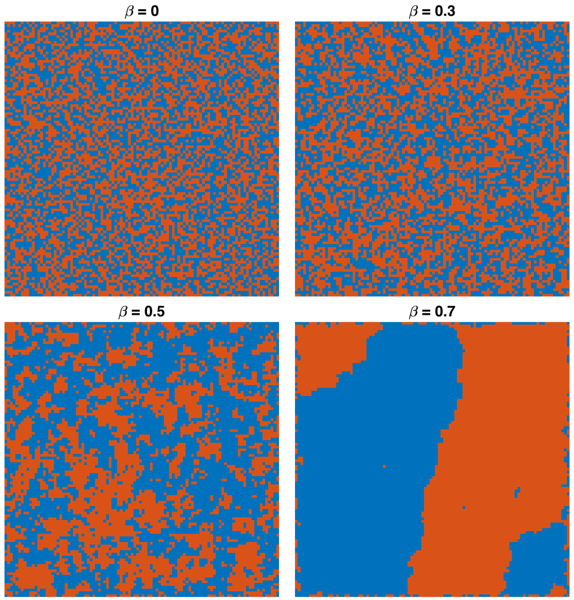
Let $\mathcal{G} = (V, E)$ be an undirected graph. A clique C of $\mathcal{G}$ is a subset of vertices such that every pair of vertices in C are adjacent.

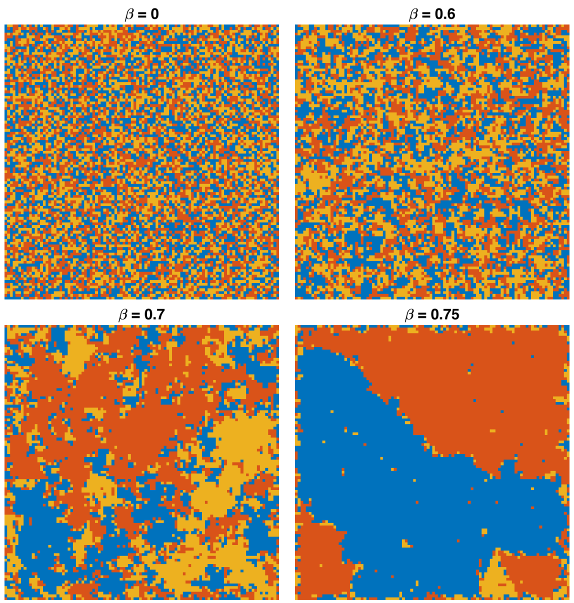
Example:



Cliques:

$$\{1\}, \{2\}, \{3\}, \{4\}, \{5\}$$
$$\{1,2\}, \{1,3\}, \{2,3\}, \{3,4\}$$
$$\{1,2,3\}$$


$\beta = 0$
$\beta = 0.3$
$\beta = 0.5$
$\beta = 0.7$


$\beta = 0$
$\beta = 0.6$
$\beta = 0.7$
$\beta = 0.75$