Statistical image analysis, 7.5hp | **Written exam**
TMS016/MSA300 | **2018-08-29, 14:00 - 18:00**

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Allowed material: Chalmers allowed calculator.

Grading: Correct and well-motivated solutions (the first question only requires an answer) give the points that are written in parentheses at each question. You can in total get 20 points on the exam, and 20 points for the project assignments. To pass the course, you must have completed all project assignments and given a project seminar. The limits for the grades are:
GU: 20 and 31 points for "G" and "VG" respectively.
CTH: 20, 28, and 34 points for 3,4, and 5 respectively.

Answers can be given in English or Swedish.

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1. Answer the following statements with one of the choices "true", "false", or "I do not know". For each statement, a correct answer gives 1 point, an incorrect answer (e.g. answering "true" when the statement is false) gives -0.5 points, and "I do now know" gives 0 points. (6p)

   (a) Figure 1 shows two covariance functions and two simulations of corresponding mean-zero Gaussian random fields. Realisation 1 is a simulation of the Gaussian field with Covariance function A.

   (b) The median filter is a linear filter.

   (c) Assume that we have an inhomogeneous Poisson process on the unit square, with intensity function
   $$\lambda(\mathbf{s}) = \lambda(s_1, s_2) = \begin{cases} 1 & \text{if } 0 \leq s_1 \leq 0.5 \\ 0.5 & \text{if } 0.5 < s_1 \leq 1. \end{cases}$$
   Then the expected number of points in the unit square is Po(2)-distributed.

   (d) A Gaussian Markov random field always has a covariance function with compact support.

   (e) Let $Y_1, \ldots, Y_n$ be observations taken at locations $\mathbf{s}_1, \ldots, \mathbf{s}_n$ of a random field $X(\mathbf{s})$. The kriging predictor $\hat{X}(\mathbf{s}_0)$ of $X(\mathbf{s})$ at some location $\mathbf{s}_0$ is always a linear combination of the observations, $\hat{X}(\mathbf{s}_0) = \sum_{i=1}^{n} \alpha_i Y_i$ for $\alpha_i \in \mathbb{R}$.

   (f) The exponential covariance function is a special case of the Matérn covariance function.

2. (a) Let S be a structure element and I a binary image. State the definitions of the erosion, dialation and opening of the image, with respect to the structure element S. (3p)

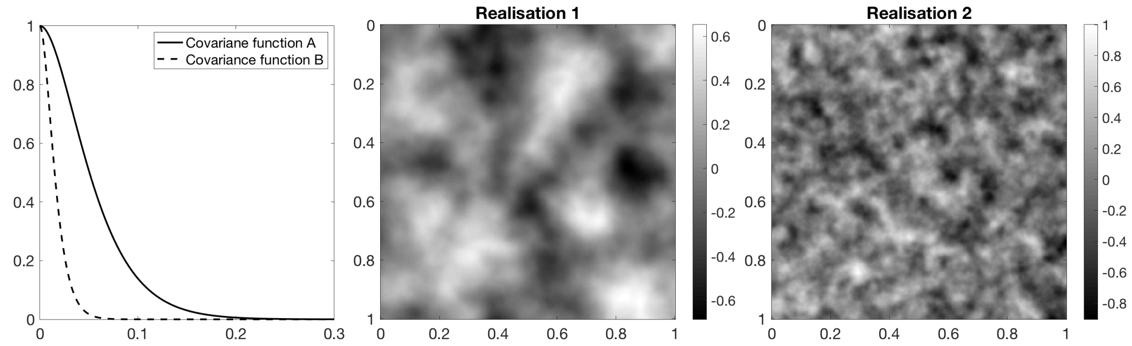

Figure 1: Two covariance functions and two simulations of corresponding mean-zero Gaussian fields.

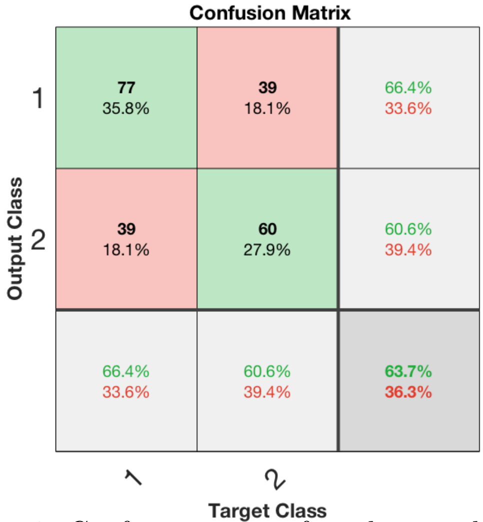

Figure 2: Confusion matrix for a binary classifier.

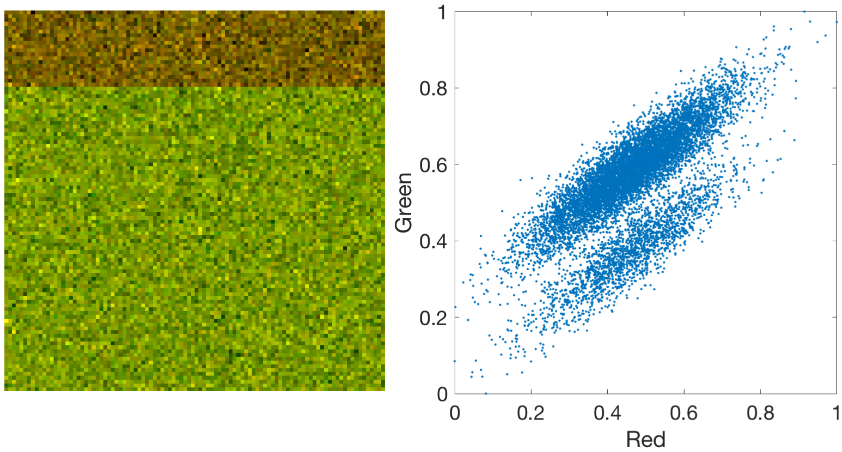

Figure 3: An RGB image without a blue component, as well as a scatter plot of the pixel values.

(b) Describe how the morphological operations in (a) can be used to remove noise from the binary image. (2p)

3. (a) When training classification models, we have two scenarios: Supervised learning and unsupervised learning. Describe these two scenarios. (2p)

   (b) A common way of visualising the performance of classifiers is to use confusion matrices. Figure 2 shows a confusion matrix for a cross-validation experiment using a binary classifier. Explain what the image is showing. Is the classifier doing a good job? (2p)

4. (a) Explain how the K-means algorithm works. (2p)

   (b) Figure 3 shows an image with only red and green colors, as well as a scatter plot of all pixel values. We clearly have two distinct regions in the image. Would the K-means algorithm work well for finding these two regions? Why/why not? (3p)

Good luck!