Spatial statistics and image analysis, 7.5hp **Written exam**
TMS016/MSA301 **2019-08-28, 14:00 - 18:00**

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Allowed material: Chalmers allowed calculator.

Grading: Correct and well-motivated solutions (the first question only requires an answer) give the points written in parentheses at each question. You can in total get 20 points on the exam, and 20 points for the project assignments. To pass the course, you must have given a project seminar, have at least 8 points on the project assignments, and 8 points on the exam. Given that this is fulfilled, the limits for the grades are:
GU: 18 and 29 points for "G" and "VG" respectively.
CTH: 18, 26, and 32 points for 3,4, and 5 respectively.

Answers can be given in English or Swedish.

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1. Answer the following statements with one of the choices "true", "false", or "I do not know". For each statement, a correct answer gives 1 point, an incorrect answer (e.g. answering "true" when the statement is false) gives -0.5 points, and "I do now know" gives 0 points. (5p)

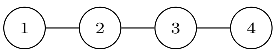
   (a) We have computed two moment features for eight different images coming from two different classes. The results are shown in the following table:

Image	1	2	3	4	5	6	7	8
Feature A	0.1	0.2	0.3	0.1	0.1	0.2	0.3	0.2
Feature B	0.1	0.2	0.1	0.2	0.4	0.3	0.3	0.4
Class	1	1	1	2	2	2	2	1

   For a new image of unknown class, we compute the features $A = 0.1$ and $B = 0.3$. If we use the KNN method to classify this image, with $K = 2$ and standard euclidian distance on $\mathbb{R}^2$, then the image gets class 1.

   (b) The semivariogram of a random field $X(\mathbf{s})$ is defined as $\gamma(\mathbf{s}_1, \mathbf{s}_2) = \mathsf{V}(X(\mathbf{s}_1) - X(\mathbf{s}_2))$.

   (c) Assume that $\mathbf{X}$ is a Gaussian Markov random field defined on the following graph:

   Then X_1 and X_4 are independent.

   (d) Figure 1 shows covariance functions and simulations of two mean-zero Gaussian random fields. Realisation 1 is a simulation of the field with Covariance function A.

   (e) The rectified linear function that is used in neural networks is defined as $g(x) = \max(0, x)$.

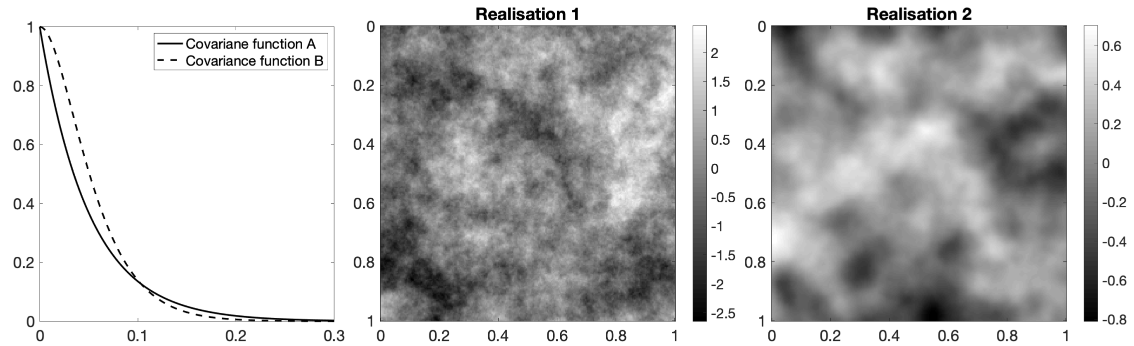


Figure 1: Two covariance functions and two simulations of corresponding mean-zero Gaussian fields.

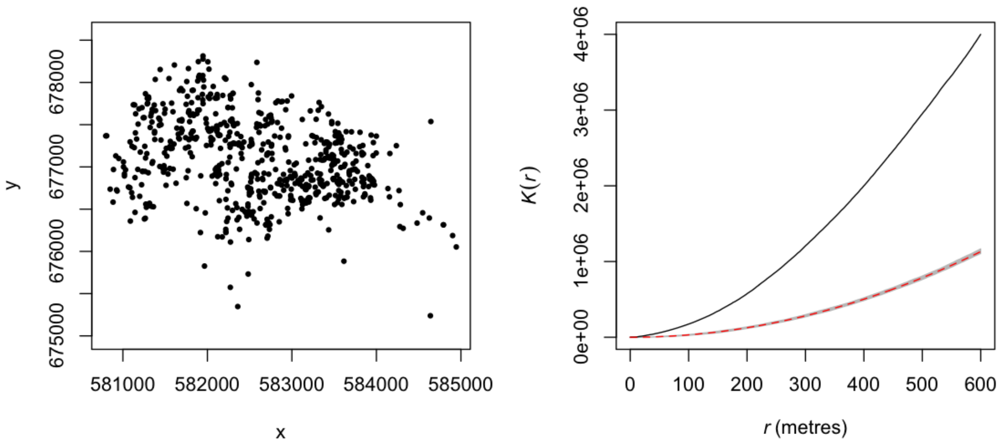


Figure 2: The left panel shows nesting sites of gorillas in the Kagwene Gorilla Sanctuary, Cameroon. The right panel shows the estimated K-function for the data (solid line) as well as the K-function for a homogeneous Poisson process (dashed lines) with associated confidence band.

2. Figure 2 shows nesting sites of gorillas in the Kagwene Gorilla Sanctuary, Cameroon, which we want to model using a homogeneous Poisson process.

   (a) Assume that the Poisson process has intensity λ. What is the probability of observing k points in a region A inside the observation domain D? (1p)

   (b) To test if the homogeneous Poisson process is a reasonable model we can use the K-function. Define the K-function for a stationary point process. (1p)

   (c) Describe a way of estimating the K-function for the data. (2p)

   (d) Figure 2 also shows the K-function for a homogeneous Poisson process as well as the estimate from the data and a confidence band. Does the model seem to fit the data? If not, would you suggest using a clustering process or a regular process instead? (1p)

3. (a) Assume that we have an image in which we want to detect vertical edges. Explain how this can be done. (2p)

   (b) Explain how a Prewitt filtered image is computed, and explain what at Prewitt filter typically is used for. (1p)

   (c) Define the moment of order (p, q) of an image x. Also define the corresponding central moment. (2p)

4. Assume that $X(\mathbf{s}), \mathbf{s} \in \mathbb{R}^2$ is a stationary Gaussian random field with mean zero and an exponential covariance function $r(h) = \exp(-h)$. Further assume that we have observed the process at two locations, $\mathbf{s}_1 = (1, 0)$ and $\mathbf{s}_2 = (1, 1)$, and got the observations $y_1 = X(\mathbf{s}_1) = 0.5$ and $y_2 = X(\mathbf{s}_2) = 1$.

   (a) Write down the formula for the semivariogram $\gamma(h)$ corresponding to $r(h)$. (1p)

   (b) Compute the kriging predictor of $X(\mathbf{s}_0)$, with $\mathbf{s}_0 = (0, 0)$, given y_1 and y_2. (2p)

   (c) Compute the variance of the kriging predictor. (2p)

Good luck!