

**TENTAMEN:** Experimental design (TMS031/MSA250)

Torsdagen den 28 april, 2011, kl 8:30-13:30

**Lärare och jour:** Aila Särkkä, tel. 772 35 42

**Hjälpmedel:** A valfri miniräknare och tabellbok (minicalculator and a book of tables).

- 1) We are comparing two entities.
  - a) What is a reference distribution?
  - b) What are the possible advantages derived from i) an external reference set and ii) a randomized reference set?
  - c) What is the value of pairing and blocking?
  - d) Why can confidence intervals be more useful than significance tests?

(8p)
- 2) A chemical engineer studied how three factors (temperature, pH and agitation rate) affect the yield of a chemical reaction by using a  $2^3$  factorial design. The design and data are as follows:

$x_1$	$x_2$	$x_3$	$y$
-	-	-	60
+	-	-	61
-	+	-	54
+	+	-	75
-	-	+	58
+	-	+	61
-	+	+	55
+	+	+	75

where

$$x_1 = \frac{\text{temperature} - 150^\circ\text{C}}{10^\circ\text{C}}, \quad x_2 = \frac{\text{pH} - 8.0}{0.5}, \quad x_3 = \frac{\text{agitation rate} - 30\text{rpm}}{5\text{rpm}}$$

and

$$y = \text{yield (\% theoretical)}.$$

Estimate the main effects and interactions. Plot the data. Explain what the results might mean, stating the assumptions you have made. (8p)

- 3) In the following table the design and the results of an experiment are presented. The 16 runs were done in random order. From experience we know that the third and higher order interactions between factors A, B, C and D can be neglected.

Run number	A	B	C	D	$y_1$	$y_2$	$\bar{y}$
1	-	-	-	-	8.34	7.47	7.91
2	+	-	-	+	5.73	5.62	5.68
3	-	+	-	+	-0.92	1.04	0.06
4	+	+	-	-	13.4	13.6	13.5
5	-	-	+	+	-1.75	-2.46	-2.11
6	+	-	+	-	10.43	9.91	10.17
7	-	+	+	-	4.71	3.56	4.14
8	+	+	+	+	3.01	3.65	3.33

- a) What is the confounding pattern?  
 b) What is the resolution of the design?  
 c) Is it possible to estimate the main effects alone?  
 d) Which factors are active? (10p)
- 4) In an experiment the amount of nickel and manganese in an alloy were changed in order to analyze the breaking strength of a component. A  $3 \times 2$  design was created with two runs in each factor combination:

Nickel (%)	Manganese (%)	Breaking strength (ft-lb)	
0	1	28	30
2	1	41	43
4	1	55	55
0	2	48	52
2	2	35	37
4	2	39	41

The alloy expert of the company believes in a regression model with a mean, two main and one interaction effect. He does not believe that quadratic terms would be significant.

- a) Formulate the model and give the assumptions needed.  
 b) Fill in the ?'s in the table below:

Source	Sum of squares	df
Model	21944	?
Residual	?	?
Total	22068	?

- c) Carry out a lack of fit test to check if the model based on the expert's advice fits the data.  
 d) Give an estimate for the error variance. (12p)

- 5) Three alternative methods (M1, M2 and M3) for reduction of weight of overweight men are compared. Fifteen volunteers were available for the trial. The trials were carried out by first dividing the subjects into “matched” groups, i.e. men in any group were chosen to be as alike as possible. The loss of weight (in pounds) after 3 months for the three methods was as follows:

Group	Method		
	M1	M2	M3
1	15	10	8
2	24	15	17
3	31	28	34
4	37	36	34
5	33	37	39

- a) Which design is this?  
 b) An incomplete ANOVA table:

Source	df	SS	MS	F value	<i>p</i> value
Method	?	19.73	?	?	?
Group	?	1507.73	?	?	?
Residual	?	92.27	?		
Total	?	1619.73			

Fill in the ?’s in the ANOVA table (it is enough to say whether the *p* value is less or greater than 0.05). Explain all the numbers in it (both those you add and those that are already there).

- c) Interpret the results in the ANOVA table.  
 d) Give the assumptions you have made.  
 e) Suppose that you are told that the average weight (in pounds) at the beginning of the trial for members in each group is as follows:

Group	1	2	3	4	5
Weight	250	309	327	356	379

How would this affect your analysis? (12p)

**Good luck!**