

Calculation of sample sizes with normal distribution as a model and known σ^2

The definitions of the percentiles of the standard normal distribution follows the book (Box, Hunter and Hunter).

One-sample case

Suppose L is the requirement on the length of a confidence interval for μ with confidence level $1 - \alpha$. Then choose n from

$$n = 4 \frac{z_{\alpha/2}^2 \sigma^2}{L^2}$$

One-sample test with $H_0 : \mu = \mu_0$ and $H_1 : \mu \neq \mu_0$, significance level α and power $1 - \beta$. The hypotheses can also be expressed as $H_0 : \Delta = 0$ and $H_1 : \Delta \neq 0$, where $\Delta = \mu - \mu_0$. Choose

$$n = \frac{(z_{\alpha/2} + z_{\beta})^2 \sigma^2}{\Delta^2}$$

Two-sample case (assuming $n_1 = n_2 = n$)

Suppose L is the requirement on the length of a confidence interval for $\mu_1 - \mu_2$ with confidence level $1 - \alpha$. Then choose n from

$$n = 8 \frac{z_{\alpha/2}^2 \sigma^2}{L^2}$$

where n is the sample size for each group.

Two-sample test with $H_0 : \mu_1 - \mu_2 = \Delta = 0$ and $H_1 : \mu_1 - \mu_2 = \Delta \neq 0$, significance level α and power $1 - \beta$. Choose

$$n = 2 \frac{(z_{\alpha/2} + z_{\beta})^2 \sigma^2}{\Delta^2}$$

where n is the sample size for each group.

Example: $\alpha = 0.05$ gives $z_{\alpha/2} = 1.96$ and $1 - \beta = 0.8$ gives $z_{\beta}(= -z_{1-\beta}) = 0.8416$.