TENTAMEN: Experimental design (March 6, 2012)

Short solutions

1) a) Let A be country, B sugar level, C water quality, D lemon flavor, and E pH value. We run the experiments suggested by a full three factor design determined by factors A, B and C. Factor D=ABC and E=AC (for example). This should give the highest possible resolution.

   b) III; D=ABC, E=AC which gives the defining relation I=ABCD=ACE=BDE

   c) $l_A = A + CE + BCD + ABDE$, $l_B = B + DE + ACD + ABCE$, $l_C = C + AE + ABD + BCDE$, $l_D = D + BE + ABC + ACDE$, and $l_E = E + AC + BD + ABCDE$

2) See the book

3) Let X_1 be the pH in the unpolluted area and X_2 the pH in the polluted area. Also, let $X_1 \sim N(\mu_1, \sigma^2)$ and $X_2 \sim N(\mu_2, \sigma^2)$. To find the common sample size, one should compute the 90% confidence interval for $\mu_1 - \mu_2$ which has the length 0.2, i.e. $\bar{X}_1 - \bar{X}_2 \pm z_{0.05}\sqrt{2}\sigma/\sqrt{n}$ ($\mathrm{Var}(\bar{X}_1 - \bar{X}_2) = 2\sigma^2/n$). Now $z_{0.05}\sqrt{2}\sigma/\sqrt{n} = 1.645 \cdot \sqrt{2} \cdot 0.4/\sqrt{n} = 0.1$ which gives $n = 87$ (nearest integer that is larger than the computed n).

4) a) 2×4 (two-way) factorial design; experimental units are pupils (or their scores); factors are gender (male-female) and schools (4 different schools)

   b) The ANOVA table becomes:

Source	df	SS	MS	F value
Gender	1	6200	6200	2.09
School	3	246726	82242	27.76
Gender:School	3	10575	3525	1.19
Residuals	32	94826	2963	
Total	39	358327		

   c) One should look at the interaction term to answer the question. $F_{0.05}(3, 32) = 2.92$ and since $1.19 < 3.32$, we cannot say that the effect of gender on the average test score is different depending on the student's school.

   d) School effect since $27.76 > F_{0.05}(3, 32) = 2.29$. No gender effect since $2.09 < F_{0.05}(1, 32) = 4.17$.

   e) Errors are independent and $N(0, \sigma^2)$-distributed.

5) a) $y_i = b_0 + b_1 x_i + e_i$

b) The ANOVA table becomes

Source	df	SS	MS	F-ratio
Model	2	4235		
Residual	13	59	4.5	
Lack of fit	3	52	17.3	24.7
Pure error	10	7	0.7	
Total	15	7		

c) Yes, there is evidence of severe lack of fit: $F_{0.01}(3, 10) = 6.55 << 24.7$. If you plot y against x you see that the relationship is not linear but quadratic. Therefore, one should fit a second-order model.

d) The MS of pure error is a good estimate for the error variance, i.e. $\hat{\sigma}^2 = 0.7$.

e) Errors are independent and $N(0, \sigma^2)$-distributed.