EXAMINATION: Tentamensskrivning i Matematisk Statistik (TMS061)

Time: Wednesday 16 January 2008

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Aid: You are allowed to use a scientific calculator and a half page (both sides) of hand written notes

Grade: You need 21 points for 5, 16 points for 4 and 11 points for 3.

Motivate all your answers. Good Luck!

1) Suppose the random variable X has a normal distribution with mean 3 and variance 9. Let $Y = \frac{1}{3}X - 1$.
a) What are the mean and variance of Y? (2p)

b) What is the probability that Y is at least 1? (1p)

2) a) Suppose that A and B are two events such that: $P(A) = 0.6$ and $P(B) = 0.8$. Are A and B disjoint? Explain. (1.5p)

b) True or false: If A and B are events, then: $P(A \cup B) \geq P(A) + P(B)$. Justify your answer. (1.5p)

3) State in your own words the Central Limit Theorem. (2p)

4) a) Someone is recording the number of clients that arrive at a shop between 3 and 4 every Saturday afternoon for three months. Which distribution best describes the recordings? (1p)

b) What are the expected value and the variance of a Poisson random variable X for which $P(X = 2) = P(X = 3)$? (1p)

5) 500 observations from a random variable X have given 35 zeros, 140 ones, 158 twos, 121 threes and 46 fours. Test using a χ^2 test the hypothesis that the random variable X is binomial with $n = 4$ and $p = 1/2$. (3p)

6) a) The random variable Z is Poisson with mean value 2.4. Compute the probability $P(Z > 2)$.(1p)

b) The random variable Y is normally distributed with mean value $\mu = 3$ and standard deviation $\sigma = 0.8$. Compute $P(Y > 2)$.(1p)

7) For the random variable X with probability density function

$$f(x) = \frac{\lambda^3 x^2}{2} e^{-\lambda x}, \quad x > 0,$$

find the maximum likelihood estimator of λ. (3p)

8) Let X be the random variable that measures the content of a bottle of a specific perfume (in ml). A sample of size 16 has been taken from the this perfume and gave $\bar{x} = 476.4$ and $s = 0.7$ ml. Assume that X is normally distributed and
a) Compute $P(X \leq 475)$. (1p)

b) Construct a confidence interval for the true mean μ for $\alpha = 0.95$. (1p).

9) Let the random variable X have the probability density function $P(X = x) = 0.1 + 0.05x, \quad x = 0, 1, 2, 3, 4$.
a) Compute $E(X)$ and $Var(X)$. (1p)

b) What is the probability $P(X_1 + X_2 > 5)$ if X_1 and X_2 are independent random variables distributed like X? (2p)

10) For a engineering study we have recorded the time it takes two different machines A and B to warm up (in min.). The results are:

$$A : 6.7 \quad 7.2 \quad 5.9 \quad 6.9 \quad 7.0 \quad 6.7 \quad 5.9$$

$$B : 5.4 \quad 5.8 \quad 6.3 \quad 6.2 \quad 5.6 \quad 5.5$$

Assume that the above observations are independent samples from a normal distribution with the same variance. Test the hypothesis that the means of the two distributions are also the same with alternative hypothesis that are different. $\alpha = 0.01$ (3p)