EXAMINATION: Tentamensskrivning i Matematisk Statistik (TMS061)

Time: Tuesday 29 May 2007

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Aid: You are allowed to use a scientific calculator and a half page (both sides) of hand written notes

Grade: You need 42 points for 5, 34 points for 4 and 26 points for 3.

Motivate all your answers. Good Luck!

1) Determine the constant c so that the following function is a probability mass function: $f(x) = cx$ for $x = 1, 2, 3, 4$. (4p)

2) Let $X_1, X_2, \ldots, X_n$ be a random sample.

a) What conditions do $X_1, X_2, \ldots, X_n$ have to satisfy? (1p)

b) If $X_1, X_2, \ldots, X_n$ are additionally normally distributed $N(\mu, \sigma^2)$ what can you say about the distribution of $\bar{X}$? (2p)

c) What is $E(\bar{X})$, $Var(\bar{X})$? (2p)

d) Is $\bar{X}$ an unbiased estimator of the true mean? (1p)

3)

$$-0.19, 0.73, 2.18, -0.14, 0.11, 1.07, 0.04, -0.10, -0.83, 0.29$$

are 10 computer generated $N(0, 1)$ observations. Test the hypothesis $\mu = 1$ with alternative $\mu \neq 1$ when

a) $\sigma = 1$ is known (3p)

b) σ is unknown (3p)

at significance level $\alpha = 0.1$.

4) Answer the following questions:

a) Can a null hypothesis be rejected at $\alpha = 0.01$ level when the p-value of the test was 0.007? (1p)

b) In an experiment if $P(A) = 0.6$ and $P(B) = 0.7$ is it possible that $P(A \cap B) = 0.2$? (2p)

c) Do X and $2 \cdot X$ have the same variance? (1p)

d) If $P(A) = 0.3, P(B) = 0.5$ and $P(A \cup B) = 0.65$, are the events A and B independent? (2p)

5) Consider the following frequency table:

Values	0	1	2	3	4	5
Observed frequency	75	140	108	66	9	2

Based on 400 observations is a binomial distribution $B(5, 0.3)$ an appropriate model? Perform a χ^2 test with $\alpha = 0.05$. (5p)

6) The performance of a mettalic device is to be tested. A sample of 14 specimen is taken to give $\bar{x} = 876.5$ and $s_x = 91.4$. After some adjustment has been performed a new sample of 16 specimen was taken to give $\bar{y} = 975.3$ and $s_y = 116.6$.

a) Formulate a suitable model to test the hypothesis that no change has occured with the introduction of the adjustment. (3p)

b) Provide with a confidence interval for the difference of the true means. (3p)

Use $\alpha = 0.05$.

7) The Rayleigh distribution has probability density function

$$f(x) = \frac{x}{\theta} e^{-\frac{x^2}{2\theta}}, \quad x > 0, \quad 0 < \theta < \infty$$

Find the maximum likelihood estimator of θ. (5p)

8) Suppose that the random variable X denotes the number of arrivals in a bus station and is distributed as a Poisson random variable with mean 4 arrivals per hour. Compute the following probabilities:

a) $P(X = 0)$ (1p)

b) $P(X \geq 2)$ (1p)

c) $P(X \leq 4 | X \geq 2)$ (2p)

d) What is the probability that the first bus arrives after one hour? (3p)

9) Assume you are asked 4 questions to which you answer by guessing. For each correct answer you give, you win 1 point and for each wrong answer you loose 1 point. Let X be the maximum between your score and zero. What is the $E(X)$? (5p)