1) $f(x) = \frac{(x-3)^2}{5}$, $x = 3, 4, 5$

a) $f(x) \geq 0, \forall x.$

$\sum_{x=3}^{5} f(x) = 1 \Rightarrow 0 + \frac{1}{5} + \frac{4}{5} = 1$ //

b) $EX = \sum_{x=3}^{5} x\, f(x) = 3 \cdot \frac{0}{5} + 4 \frac{1}{5} + 5 \cdot \frac{4}{5} = \frac{4+20}{5} =$

$= \frac{24}{5}$

$\mathrm{Var}\, X = \sum_{x=3}^{5} (x - EX)^2 \cdot f(x) = \left(3 - \frac{24}{5}\right)^2 \cdot 0 + \left(4 - \frac{24}{5}\right)^2 \cdot \frac{1}{5}$

$+ \left(5 - \frac{24}{5}\right)^2 \cdot \frac{4}{5} = 0 + \frac{16}{25} \cdot \frac{1}{5} + \frac{1}{25} \cdot \frac{4}{5} = \frac{16+4}{5 \cdot 25} =$

$= \frac{20}{5 \cdot 25} = \frac{4}{25}$ //

2) $X, Y \sim U(1,3)$

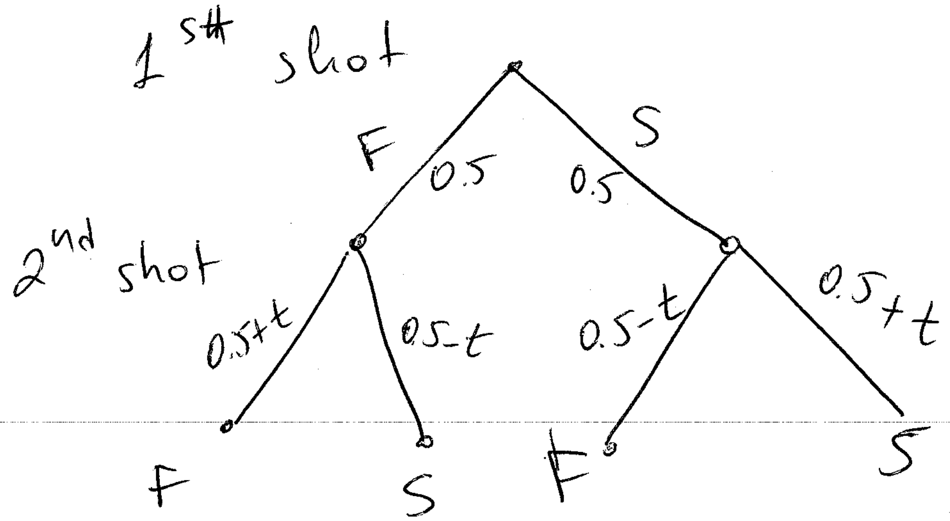
$A = \frac{X \cdot Y}{2}$

X,Y independent

$E(A) = E\left(\frac{X \cdot Y}{2}\right) = \frac{1}{2} E(X \cdot Y) = \frac{1}{2} EX \cdot EY.$

Finally, $\operatorname{Var} A = EA^2 - (EA)^2 = \frac{169}{36} - 2^2 = \frac{25}{36}$ //

3)



$$\rho_{X_1,X_2} = \frac{\operatorname{Cov}(X_1,X_2)}{\sqrt{\operatorname{Var} X_1 \cdot \operatorname{Var} X_2}} = \frac{EX_1 \cdot X_2 - EX_1 \cdot EX_2}{\sqrt{\operatorname{Var} X_1 \cdot \operatorname{Var} X_2}}.$$

$EX_1 = 1 * 0.5 + 0 * 0.5 = 0.5.$

$P(X_2 = 1) = 0.5 \cdot (0.5 + t) + 0.5(0.5 - t) = 0.5$ see picture

$EX_2 = 1 \cdot 0.5 + 0 \cdot 0.5 = 0.5.$

$\operatorname{Var} X_1 = EX_1^2 - (EX_1)^2.$

$EX_1^2 = 1^2 \cdot 0.5 + 0^2 \cdot 0.5 = 0.5 \Rightarrow \operatorname{Var} X_1 = 0.5 - 0.5^2 = \underline{\underline{0.25}}$

$EX_2^2 = 1^2 \cdot 0.5 + 0^2 \cdot 0.5 = 0.5 \Rightarrow \operatorname{Var} X_2 = \quad - \quad - \quad = 0.25.$

$EX_1 X_2 = 1 \cdot P(X_1 = 1, X_2 = 1) = P(X_1 = 1) \cdot P(X_2 = 1 \mid X_1 = 1) =$

$= 0.5 * (0.5 + t) = 0.25 + 0.5t.$

5) X = # of cars whose breaks need relining. (5)

a) $X \sim B(n,p) = B(100, 0.1)$.

$EX = np = 100 * 0.1 = 10$

b) Normal approximation to binomial is valid when $np, n(1-p) \geq 5$, which is satisfied here.

c) $P(X \geq 17) = \,?$

$$X \sim B(n,p) \Rightarrow Z = \frac{X-np}{\sqrt{np(1-p)}} \sim N(0,1)$$

$$\therefore P(X \geq 17) = P\left(Z \geq \frac{17-np-0.5}{\sqrt{np(1-p)}}\right) =$$

$$= P\left(Z \geq \frac{17-10-0.5}{\sqrt{90 * 0.1}}\right) = P\left(Z \geq \frac{6.5}{3}\right) \approx P(Z \geq 2.17)$$

$$= 1 - P(Z < 2.17) = 1 - 0.984997 \simeq 1 - 0.985 = \underline{\underline{0.015}}$$

7 . $n=15, \quad \bar{x}=11.3, \quad s=1.28$ (7

a) $H_0: \mu=10$ $\qquad \alpha=0.01$.

$H_1: \mu \neq 10$

$$t=\frac{\bar{x}-\mu_0}{\sqrt{s/n}}=\frac{11.3-10}{\sqrt{1.28/15}}=\frac{1.3}{0.2921} \approx 4.45$$

$t_{0.05,14}=1.761$.

Since $t_0=4.45>1.761=t_{0.05,14}$ we reject H_0.

b) $X^2=\frac{(n-1)S^2}{\sigma^2} \sim \chi^2(n-1)$ if $X_1, X_2 .. X_n \sim N(\mu, \sigma^2)$

$\therefore$ A $100(1-\alpha)\%$ CI for σ^2 is

$$\frac{(n-1)s^2}{\chi^2_{\alpha/2, n-1}} \leq \sigma^2 \leq \frac{(n-1)s^2}{\chi^2_{1-\alpha/2, n-1}}, \quad \chi^2_{0.005,14}=31.32$$

$$\sigma^2 \in \left[\frac{14 * 1.28^2}{31.32}, \frac{14 * 1.28^2}{4{,}07}\right]=[0.7324, 5.6358].$$

$\underline{\underline{\sigma \in [0.8558, 2.3740]}}$

b) $Var(Y_i) = \sigma^2$

$$\hat{\sigma}^2 = \frac{SSE}{n-2} = \frac{SSTotal - \hat{\beta}_1 \cdot S_{xy}}{n-2} =$$

$$= \frac{\sum y_i^2 - n \cdot \bar{y}^2 - 1.5 * S_{xy}}{n-2}$$

$$S_{xy} = \sum x_i y_i - \frac{\sum x_i \sum y_i}{10} = 285.625 - \frac{16.75 * 170}{10} = 0{,}875$$

$$\therefore \hat{\sigma}^2 = \frac{2898 - 10 \cdot \left(\frac{170}{10}\right)^2 - 1.5 * 0.875}{8} = \frac{2898 - 2890 - 1.3125}{8}$$

$$= \boxed{0.8359 = \hat{\sigma}^2}$$

c) $H_0: \beta_1 = 0$ $\qquad \alpha = 0.01$.

$H_1: \beta_1 \neq 0$

$$T = \frac{1.5 - 0}{\sqrt{\frac{0.8359}{25.8344}}} = \frac{1.5}{0.1799} = 8.339.$$

$$S_{xx} = \sum x_i^2 - \frac{\left(\sum x_i\right)^2}{n} = 28.64 - \left(\frac{16.75}{10}\right)^2 = 25.8344$$